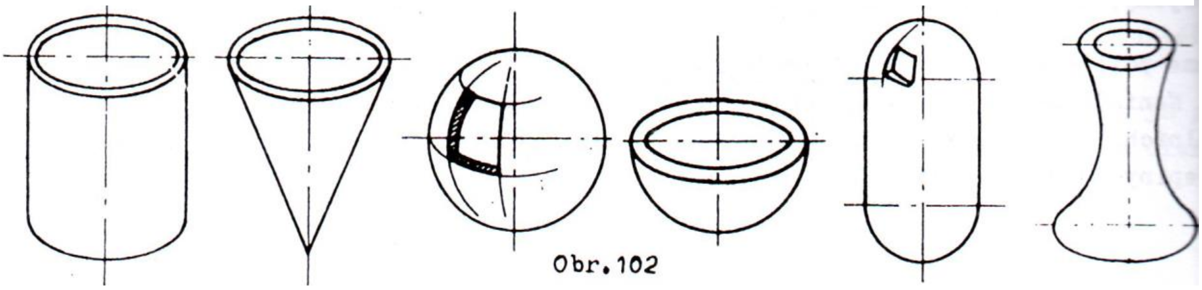
Axisymmetric membrane shell

is a thin-wall body, defined by its middle area and the thickness being substantially smaller than the other dimensions. The middle area is created by rotation of the meridian curve around an axis which becomes thus the axis of shell symmetry. The presented linear theory enables to calculate stresses in a shell; however, under negative stresses (overpressure from outside) **buckling may occur** (a non-linear behaviour tending to shell failure) which cannot be predicted on the basis of the calculated stress values!

Examples of axisymmetric shells:

cylindrical conical spherical semispherical toroidal general



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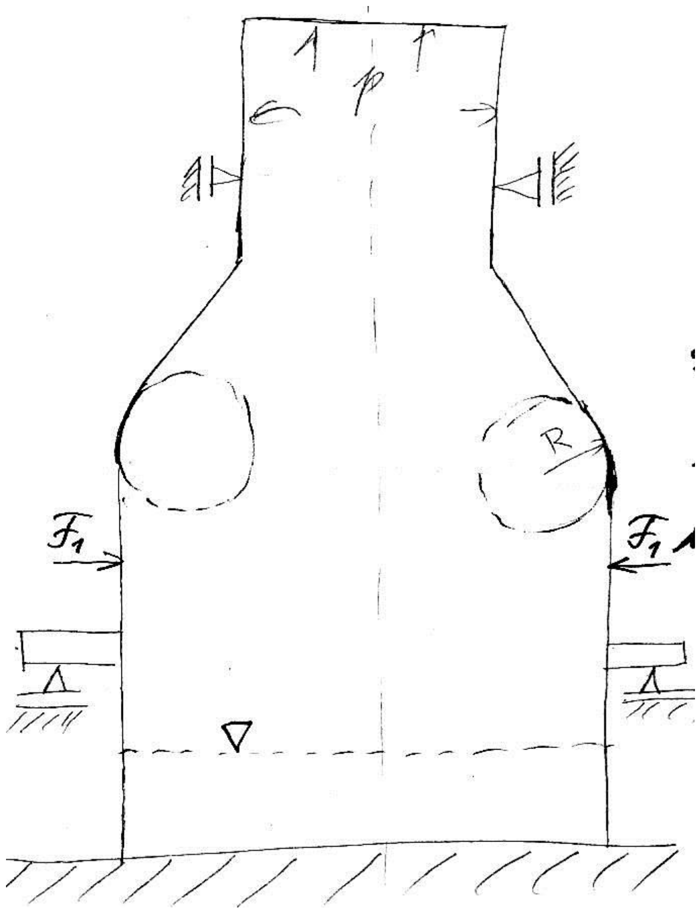
Membrane theory of shells

is based on the assumption the shell can bear only membrane forces (in-plane forces, acting in a tangential plane of the shell); the induced stresses are assumed to be uniformly distributed throughout the shell thickness (the stress state is biaxial, called **membrane stress state**).
Advantage: higher load-bearing capacity, because the shell stiffness in tension is on higher order than in bending.

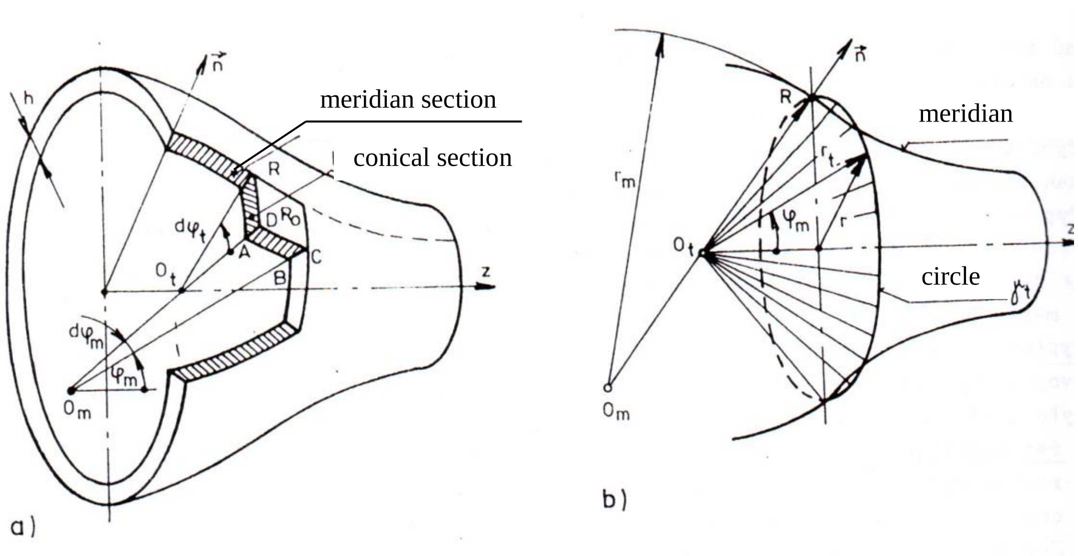
To achieve the membrane stress state, a sophisticated design is necessary; it must meet the following requirements (ordered below with decreasing importance):

1) Isolated (concentrated) external forces (incl. support reactions) must not have any components perpendicular to the middle area, i.e. must act in its tangential directions.
2) Radial deformation of the shell must not be constrained.
3) The shell thickness must not show any stepwise changes.
4) Distribution of loads acting perpendicularly to the shell surface should be continuous and smooth.
5) The middle area should be smooth and also without any stepwise changes in its curvature.

See examples of possible violations of membrane stress state in the figure.



A typical element is two times infinitesimal, created by two meridian and two conical sections.



r_t – conical radius of curvature (curvature of the conical section, length of the normal line of the middle area up to the point O_t of its intersection with the axis of symmetry)
O_t – centre of curvature of the conical section (on the axis of symmetry)

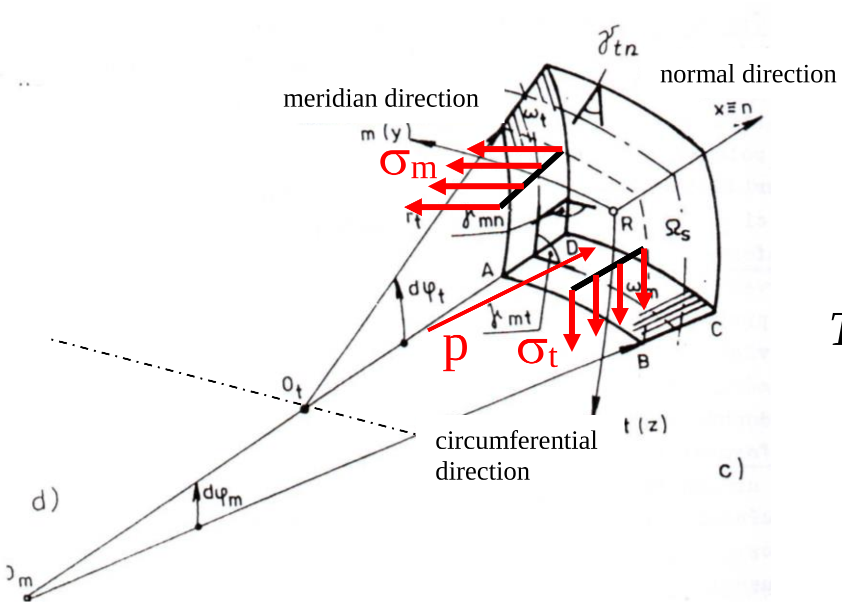
$$\frac{r}{r_t} = \sin\varphi_m$$

φ_m – top angle of the conical section

r_m – meridian radius of curvature (curvature of the meridian section)
O_m – centre of curvature of the meridian section (usually not lying on the axis of symmetry)

The circumferential direction (tangent to the circle) and the meridian direction (tangent to the meridian) are two **principal directions of curvature**.

Free body diagram and equilibrium of a shell element



$$T_\sigma = \begin{bmatrix} \sigma_m & 0 & 0 \\ 0 & \sigma_t & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The equation of static equilibrium (of forces in the normal direction) results in the **Laplace equation** (or Laplace law)

$$\frac{\sigma_t}{r_t} + \frac{\sigma_m}{r_m} = \frac{p}{h}$$

The second equation needed for solution to the unknown stress components is the **equation of static equilibrium** of a finite shell element (separated by a conical section) for force components in the **axial (*z*) direction**. Its general formulation can be as follows:

$$2\pi r h \sigma_m \sin\varphi_m - F_z + F_{rz} = 0$$

Here φ_m is the top angle of the conical section (see figure b) above).

The specific shape of this equation depends on the loads and supports of the investigated shell. The force resultant of external loads in the axial direction F_z consists of:

1) force resultant of pressure in the plane of the section,
2) weight of the shell contents,
3) weight of the shell element itself (often negligible against the weight of the contents).

F_{rz} is the sum of axial components of support reactions acting on the shell element.

Note 1: If the gravitational forces are not negligible the shell axis must be oriented vertically for not to violate the axisymmetry of the shell load.

Note 2: High axial loads may occur in pipelines (cylindrical shells) under temperature changes due to heat expansion. If this is the case, they should be reduced by using compensating (bended) pipes.

Typical solutions:

1. Constant pressure p - hydraulic, substantially higher than the hydrostatic pressure and the gravitational forces.
2. Hydrostatic pressure – changing linearly in vertical direction.

Typical most frequent shapes of the meridian:

a) straight line $\rightarrow$ cylinder or cone

b) circle $\rightarrow$ sphere

Combination of a cylindrical and spherical shell

Radial displacements of a cylinder and sphere (calculated from the circumferential strain component) do not equal to each other for the same radius $\rightarrow$ constraint of their radial displacements disturbs the membrane stress state in the location where both shells are joined together.