evropský sociální fond v ČR
EVROPSKÁ UNIE
MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY
OP Vzdělávání pro konkurenceschopnost
INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

Experimental methods in mechanics of solids

Role of experiments in stress-strain analyses and in assessment of failure risk:

- **to acquire input data** for computations (material characteristics, limit values of the relevant quantities, assessment of the character and magnitude of loads),
- **to verify** the applied theories and algorithms and to **validate** the results of computational models,
- to detect **cracks** and monitor their behaviour,
- **to obtain** some results in the fields of problems where no computational modelling is possible (problems of abrasion, corrosion, erosion, cavitation, pitting, etc.).

Additionally, the role of experiments is unavoidable in research.

Conclusion:

- **no computation can exist without experiment,**
- only very simple experiments can exist without computations.

Systemization of measuring methods in mechanics of solids

- methods for evaluation of **stresses and strains,**
- methods for monitoring of the **fracture process,**
- methods for evaluation of **body movements,** incl. its distortions (**displacements**),
- methods for evaluation of **external loads** acting upon bodies.

Warning! It is impossible to measure stresses directly!!!

Evaluation of stresses is **always based on calculations**:

1. Strains can be measured directly or calculated from the measured displacements. For the calculation of stresses, knowledge of **constitutive relations and their parameters is necessary.**
2. To obtain the constitutive relations (between stresses and strains) and their parameters, some **basic mechanical tests** are necessary. In these tests the stress values are calculated on the basis of the **measured force** and some **assumptions** on the stress distribution (uniaxial tension, bending, etc.).

Consequently you are capable to evaluate stresses **in linear elastic materials only**.

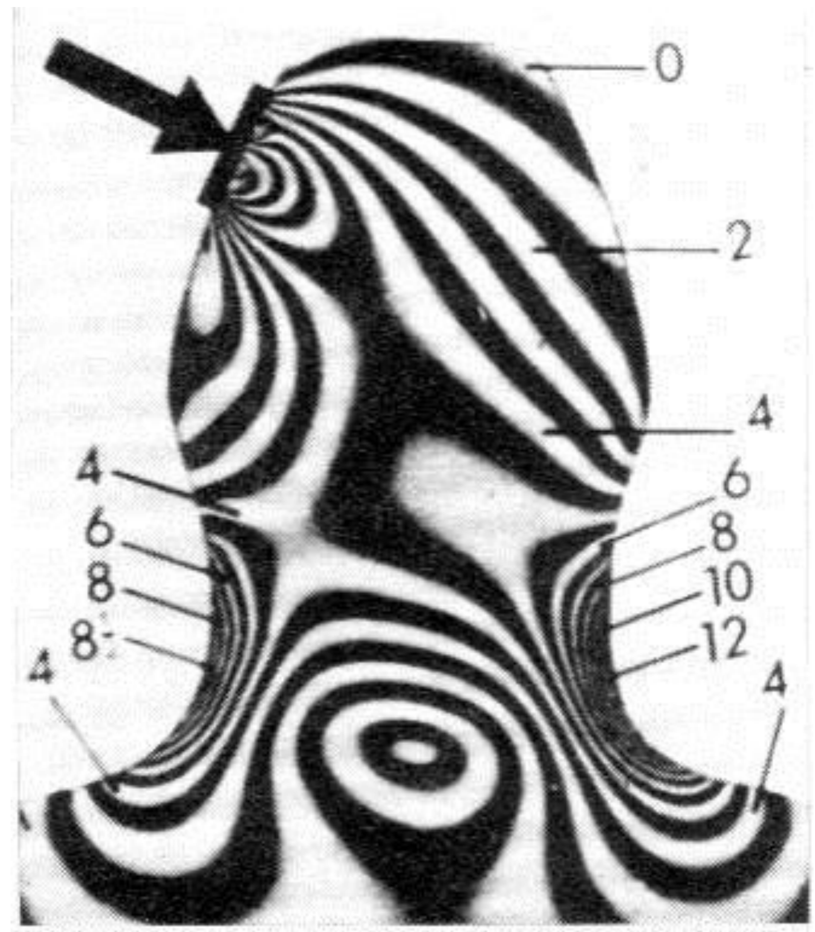
Overview of basic methods for stress and strain experimental evaluation

- **Strain gauges** – the most frequent method in practical applications. In contrast to other methods it is local, it does not measure a strain field (strains in the whole body or on its whole surface) but only the change of a specified length, which is recalculated into the average strain value specific for the gauge location. Therefore the accuracy depends on the gauge size and on the strain gradient.
- **Photoelasticimetry** – a complex experiment with a transparent model using polarized light. It is based on the photoelastic effect: some transparent materials become optically anisotropic under load.
- **Brittle laquers** – based on the low ultimate strain of some resins which crack at values of strains lower than yield strains for metals. These lacquers (or films) indicating strain above a certain limit are advantageous for finding dangerous locations of the body and the directions of the maximum principal strain (stress) here.
- **Moiré** method – is based on the light interference when passing diffraction lattices; the difference between the deformed and reference lattices creates Moiré strips corresponding to displacements equal to the lattice span.
- **Holografic methods -** based on the laser light interference between the hologram of an undeformed body and the real deformed body. The created interference strips are proportional to the displacement magnitude. Disadvantage – an accurate mutual positioning of the hologram and the body is extremely difficult.
- **Radiographic strain measuring** – is based on the diffraction of a monochromatic X-radiation on the crystallic lattice (with span on the order 10^{-10} m), which acts as a diffraction lattice.
- **Stress Pattern Analysis** (SPATE method) – it exploits the transformation of the strain energy into heat. It evaluates temperatures in different points of the body under conditions of repeated deformation caused by cyclic loading.

Photoelasticimetric image of a gear tooth

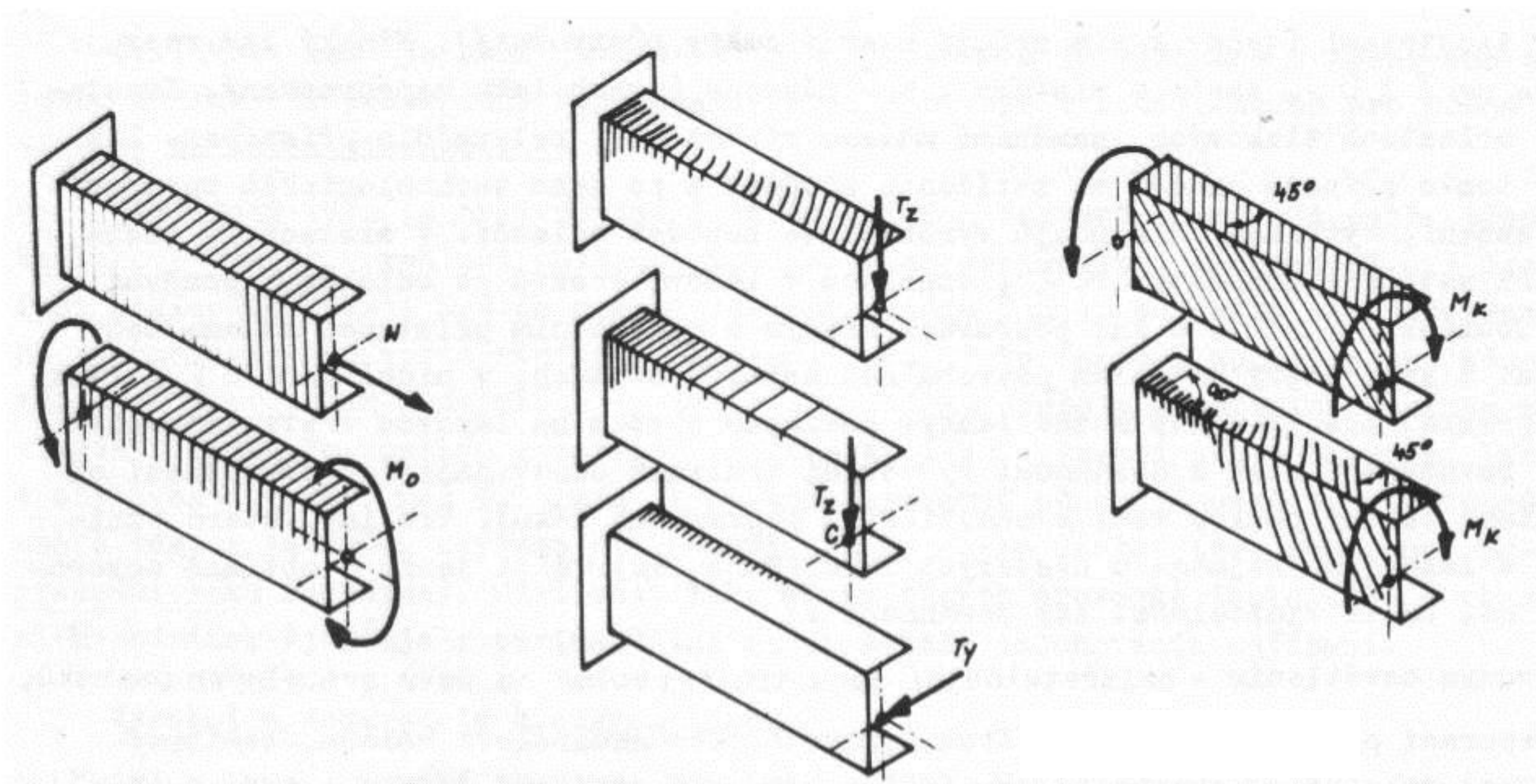
Isochromats

– lines connecting points with an identical difference between principal stresses



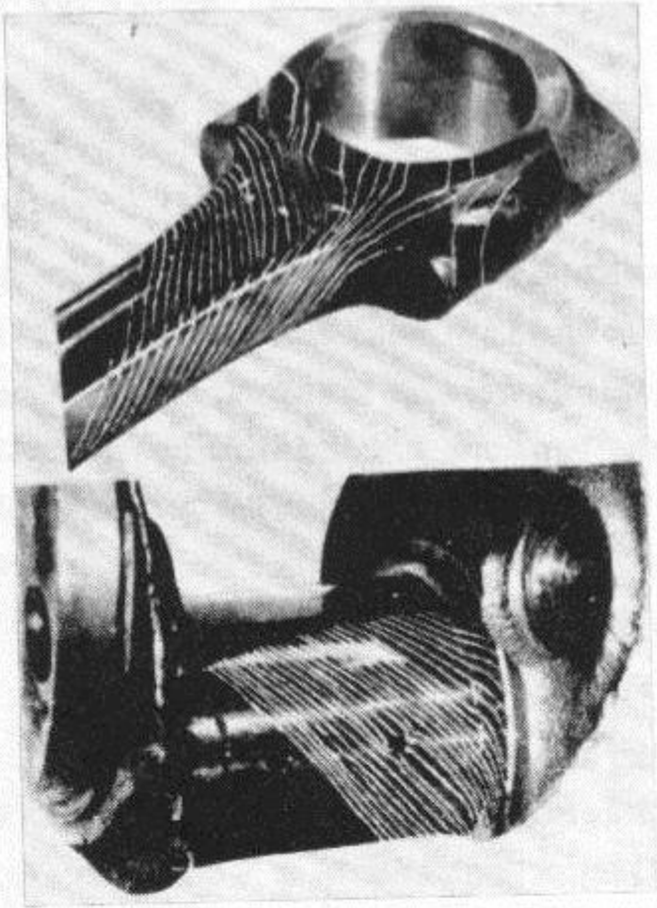
Examples of application of **brittle laquers**

with U-shaped profiles under various loads

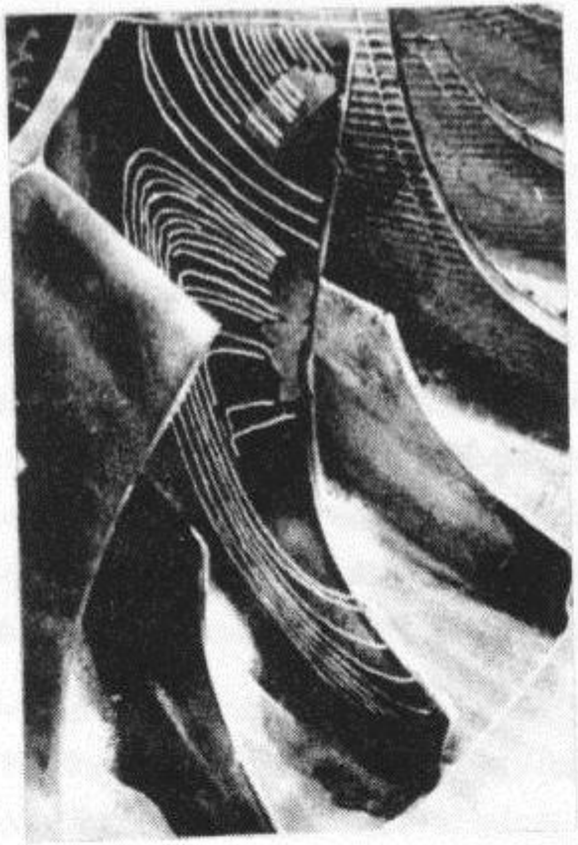


Examples of application of **brittle laquers**

Piston rod and bearing pin under torsion

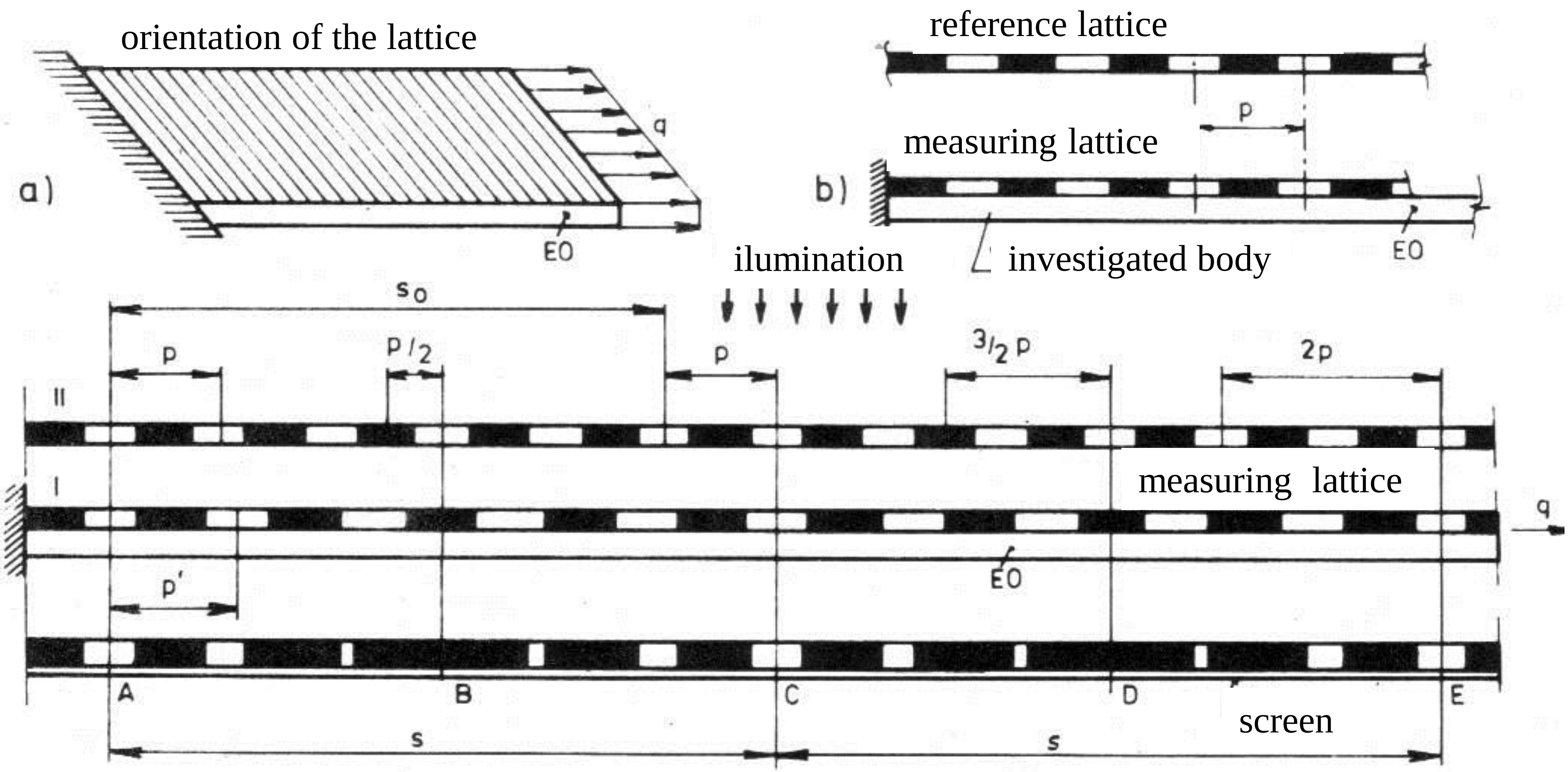


Vanes of a compressor impeller wheel



Principle of moiré method

Longitudinal engineering strain $\varepsilon_x = \dfrac{p}{s_0} = \dfrac{p}{s-p}$



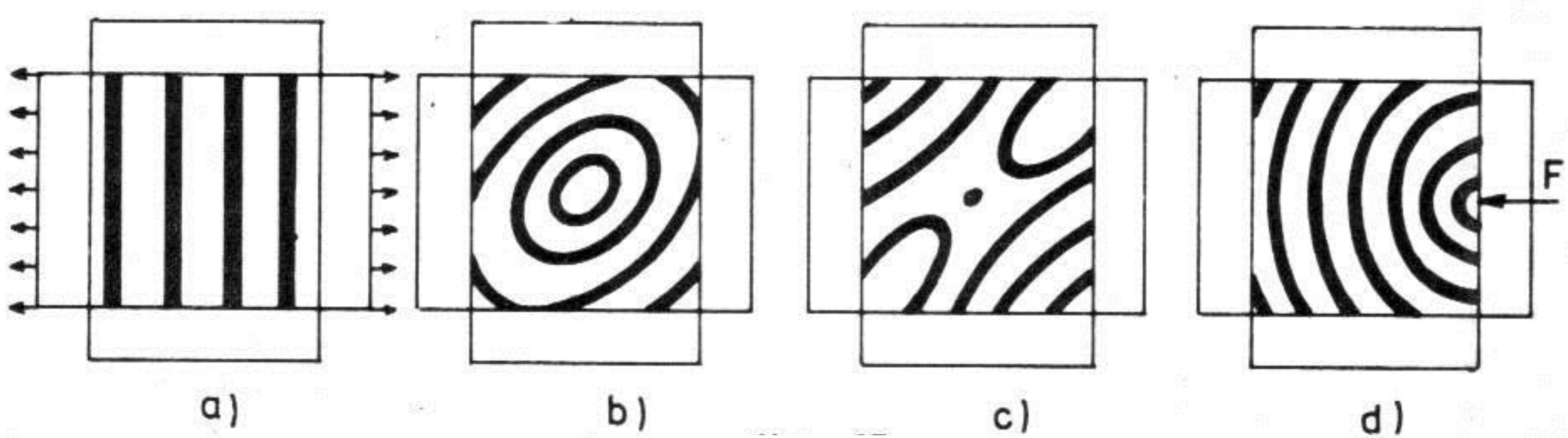
Examples of simple moiré strips
(contour lines)

Uniform uniaxial tension

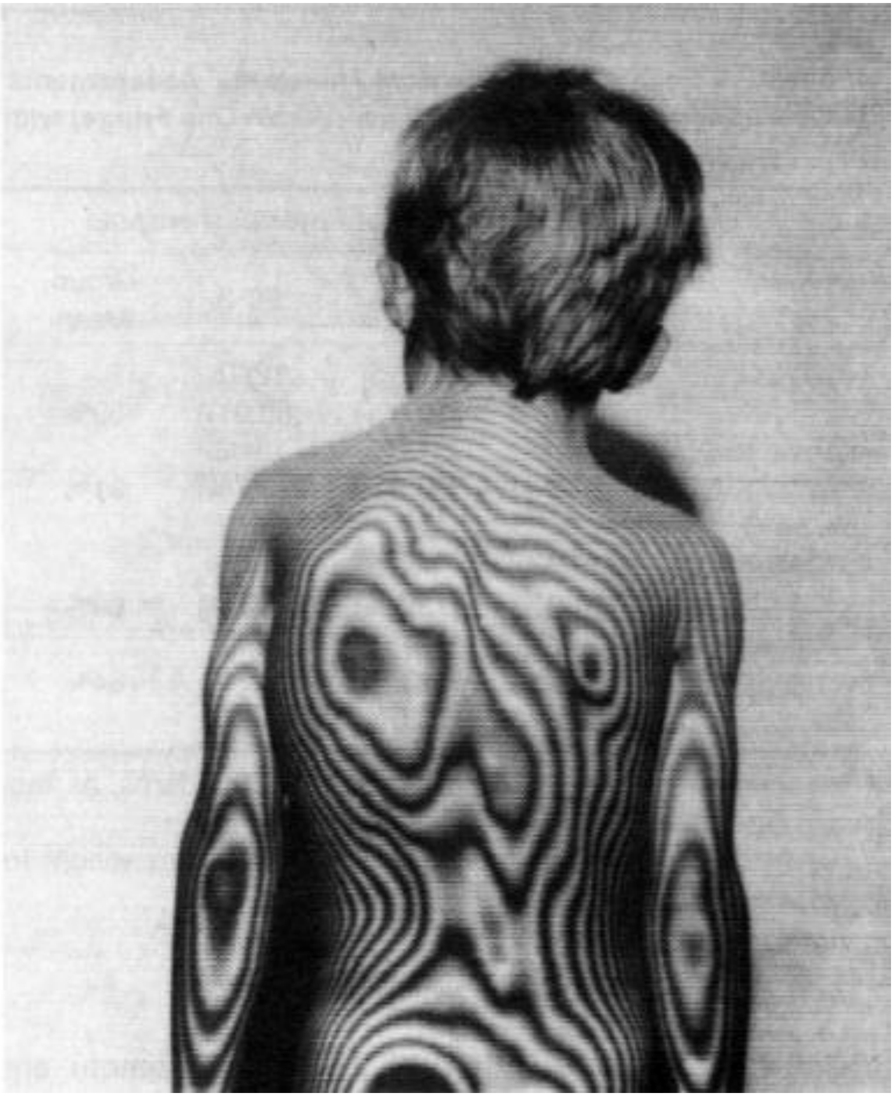
Local maximum of displacements

Local minimum of displacements

Loading by an isolated force



Application of moiré method in medicine



Holografic interferometry

In real time application, this method is based on interference of a laser beam between the hologram of the undeformed body and the real deformed body.
The created interference strips are proportional to the displacement magnitudes.
Measuring sensitivity is comparable with the wavelength of the applied laser (10^{-7} m).
The interference strips in the figure are created by vibrations of a circular plate.

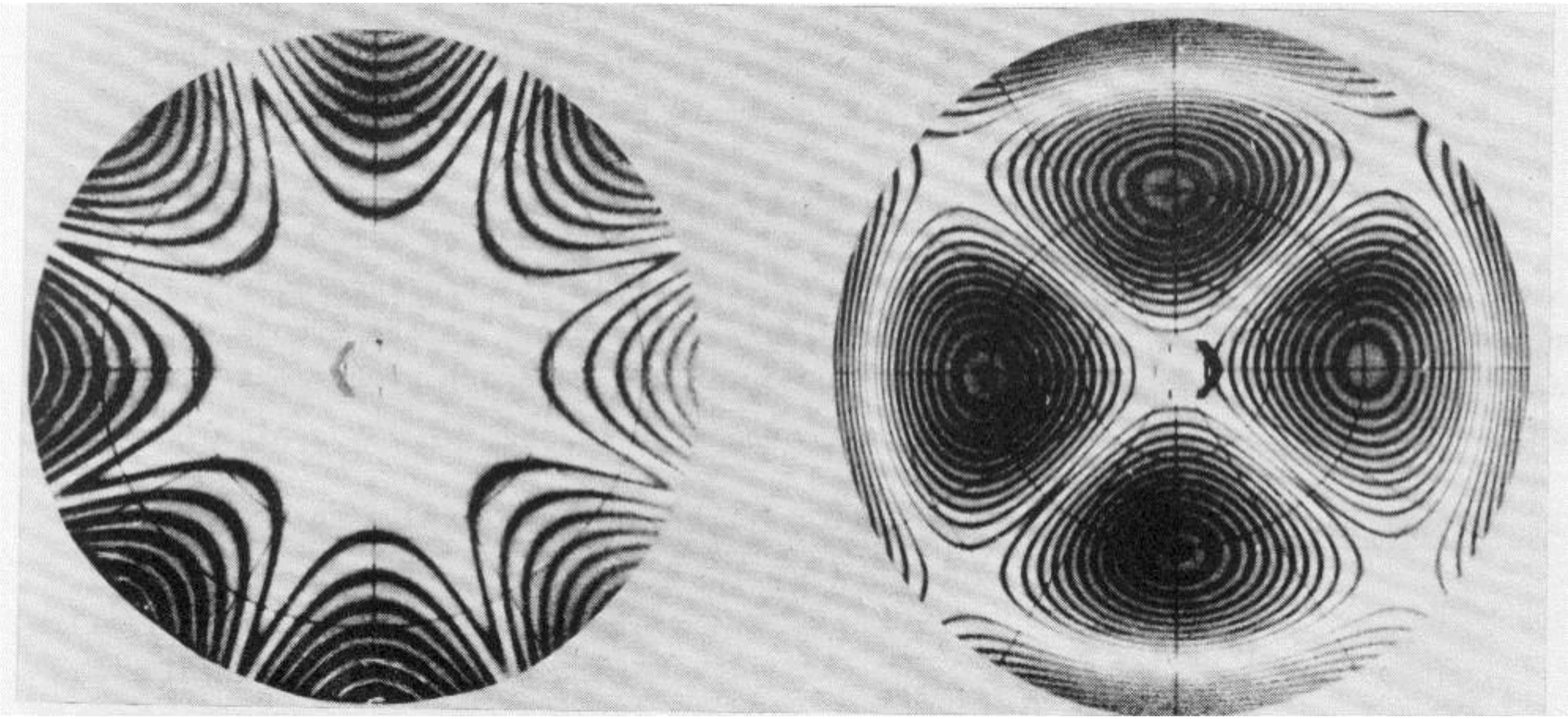
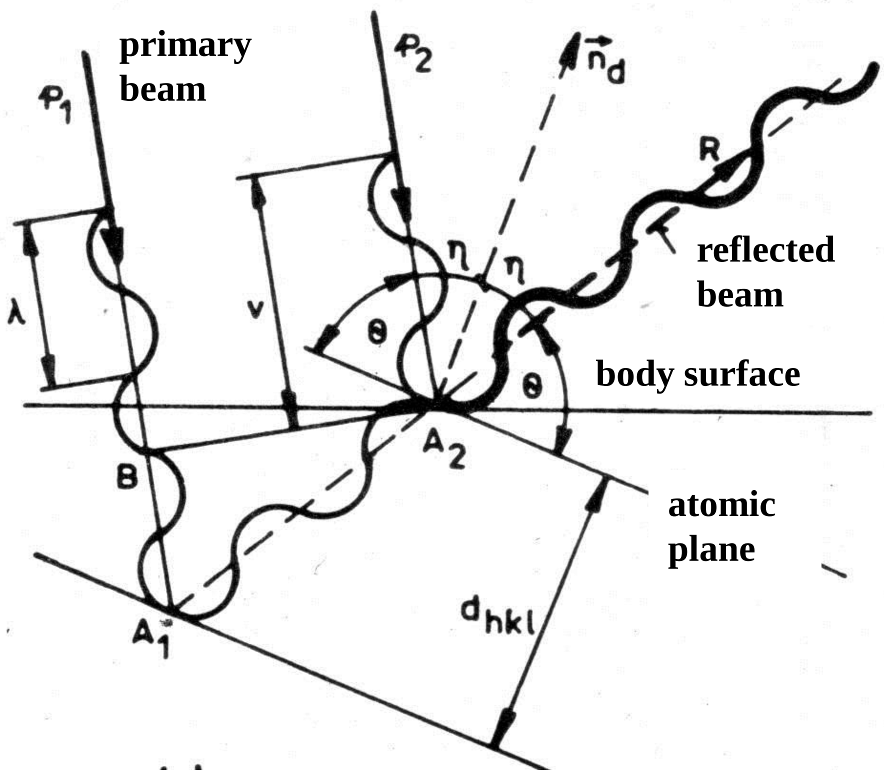


Plate supported in its centre Plate supported in 4 points on the circumference

Radiographic strain measuring

The diffraction of the X-radiation (wave length $10^{-8} - 10^{-11}$ m) can be interpreted as a reflection of beams by crystalographic planes, which acts as a diffraction lattice. Spots with maximal and minimal intensity of the reflected radiation depend on their distance; when these distances (with their span on the order of 10^{-10} m) are changed by the load, a change in diffraction images occurs.



Types of strain gauges

- **Mechanical strain gauges** – they use sharp contact edges or tips, basement length typically 10^1 mm. Used e.g. as extensometers in uniaxial tension tests.
- **String strain gauges** – change in the string tension changes the frequency of string vibrations, basement lengths of 50-400 mm, applications mostly in civil engineering (dams).
- **Pneumatic strain gauges** – used sharp contact edges are fixed to a jet and flap (with mutual distance of 10^0 mm); their distance influences the air pressure on the entrance of the jet.
- **Electric strain gauges**
  - **resistance strain gauges –** change of electric resistance of the sensor as a consequence of changes in its length, the **most frequent in technical practice**,
  - **semiconductor strain gauges** – based on the piezoresistive effect, more sensitive change of electric resistance,
  - **inductivity strain gauges** – basement length tens of mm or more, the change in position is transformed into a change of inductivity,
  - **capacitor strain gauges -** the change in position is transformed into a change of the capacity of a capacitor.

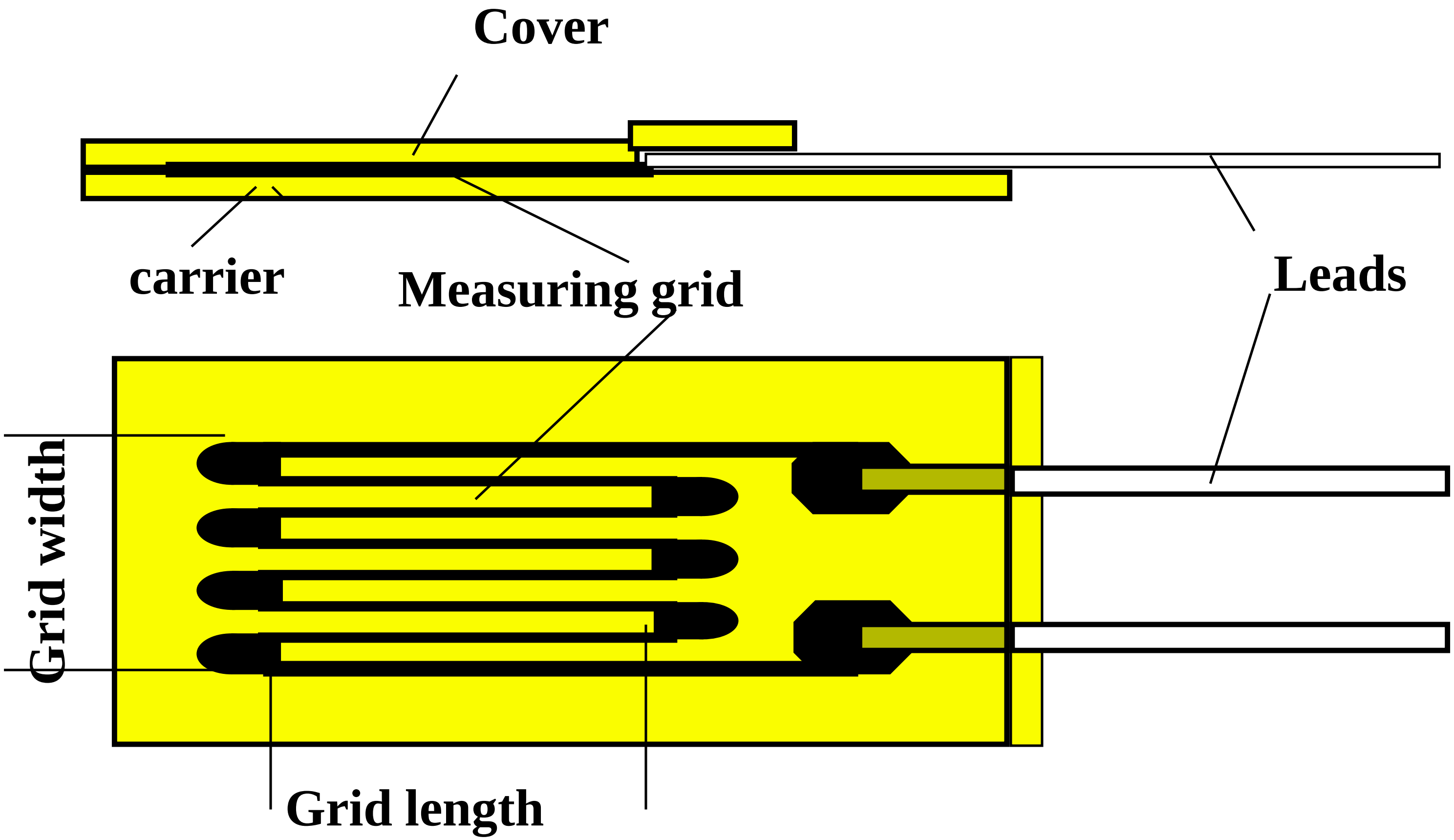
Inductivity strain gauge

Change of distance (on the order of 10^2 mm) between two points is transformed into change of inductivity of the coil.

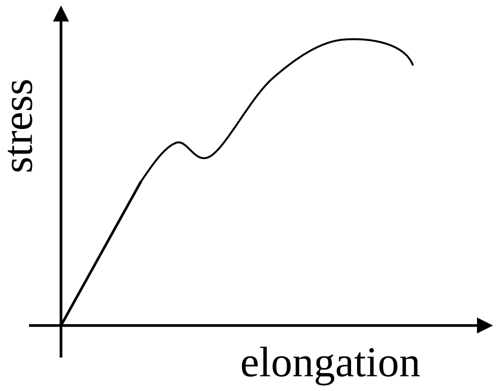
Aplication in civil engineering for checking distortions inside a dam (Kružberk, northern Moravia).

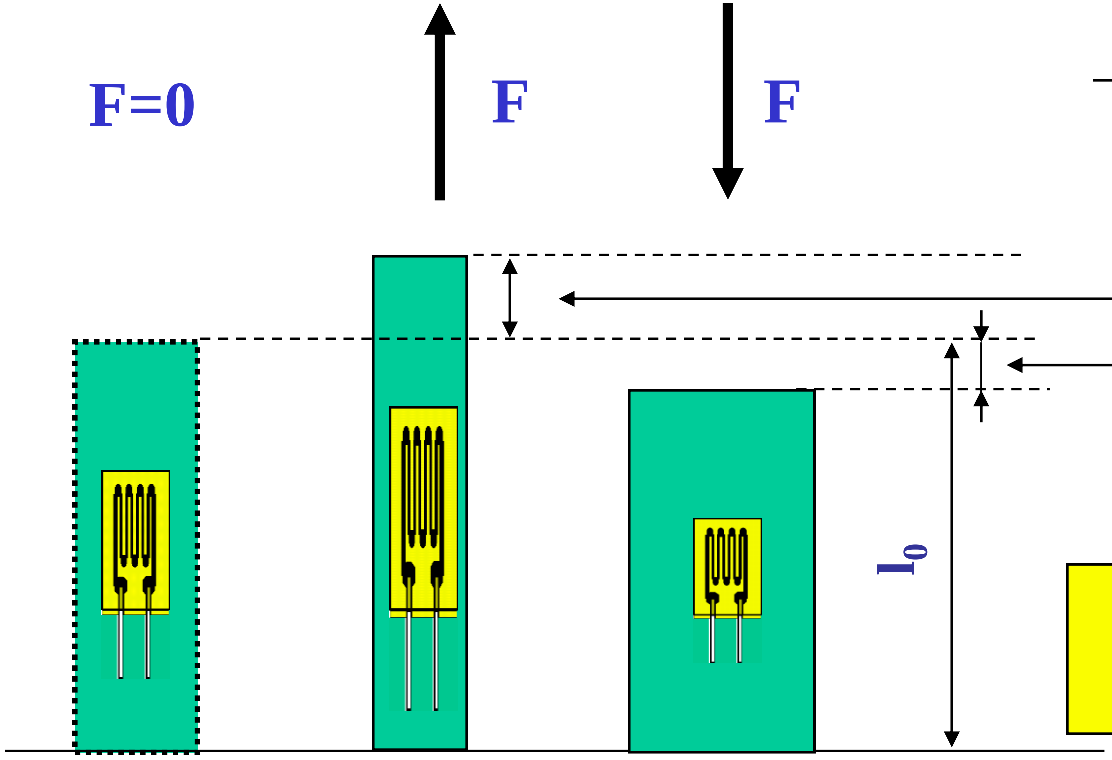
Electric resistance strain gauges

The evaluated strain is **average value** along the grid length and width. Therefore small gauges should be applied in locations with highly varying strain values (high gradients of strains and stresses).



Principle:





$$\varepsilon = \Delta l / l_0$$

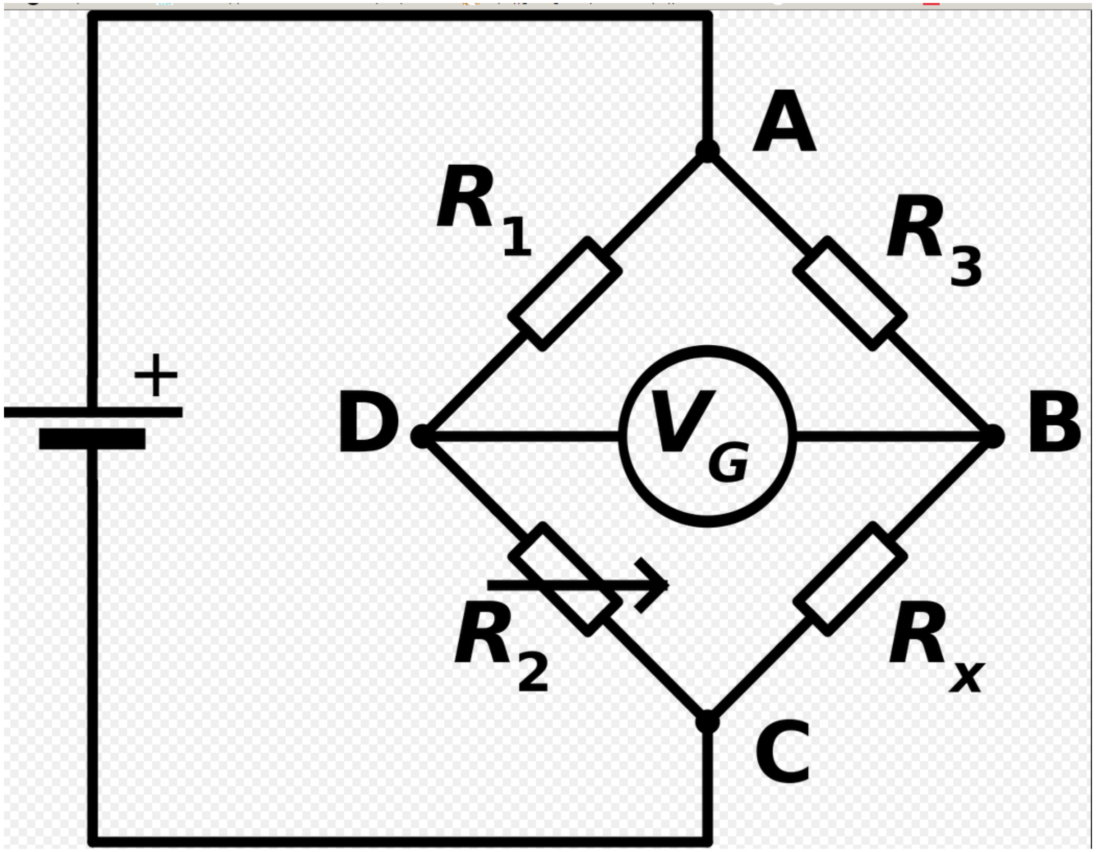
Strain measurement

Gauge factor **k** defines the **sensitivity** of the gauge

$$\Delta R/Ro = k * \varepsilon$$

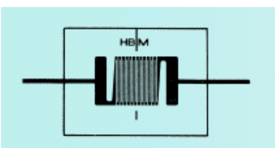
A strain gauge converts mechanical strain into a change of the gauge electrical resistence (measured by means of Wheatstone bridge). The sensitivity can be increased by using several gauges in the same bridge. If we use the same resistor magnitudes in the bridge, it holds:

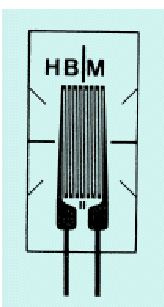
$$V_G = \frac{\Delta R_2 - \Delta R_x}{R_1 + R_2} V_{source}$$



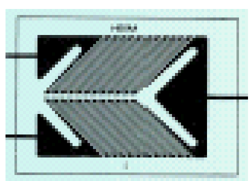
Basic types of strain gauges

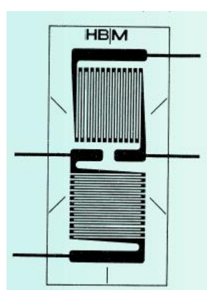
Uniaxial gauge
(for uniaxial stress)



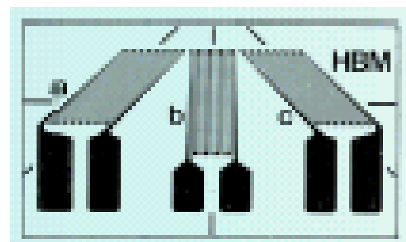


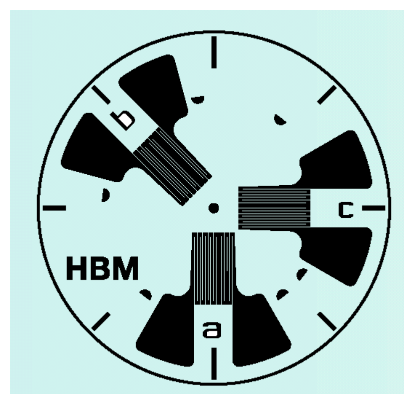
T-rosette
(right gauge - for biaxial stresses with known principal directions)



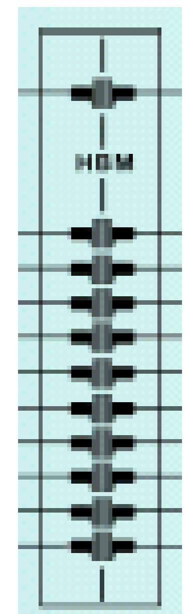


Rosette
(for biaxial stresses with unknown principal directions)





Strain measuring chain



Calculation of principal stresses for strain values measured by a rosette

On the basis of Mohr's representation of strain tensor, the following formula can be derived for calculation of principal strains from the three strain values measured in three consequent directions A, B, C rotated mutually by 45°:

$$\varepsilon_{I,II} = \frac{\varepsilon_A + \varepsilon_C}{2} \pm \sqrt{\left(\frac{\varepsilon_A - \varepsilon_C}{2}\right)^2 + \left(\frac{\varepsilon_A - 2\varepsilon_B + \varepsilon_C}{2}\right)^2}$$

Their recalculation into principal stresses must be done (on the basis of assumption of linear elastic isotropic material) by means of Hooke's law for planar stress state (stresses perpendicular to the measured surface are zero).

$$\sigma_I = \frac{E}{1-\mu^2}\left(\varepsilon_I + \mu\varepsilon_{II}\right) \qquad \sigma_{II} = \frac{E}{1-\mu^2}\left(\varepsilon_{II} + \mu\varepsilon_I\right)$$

The same form of Hooke's law is used for calculation of stresses in case of application of a T-rosette (oriented in principal strain directions).

Application limited to **linear elastic isotropic materials**.

Methods for monitoring the fracture process

The most frequent methods for monitoring the fracture process are as follows:

- **Acoustic emission ─ detection by piezoelectric sensors**
- Crack detection by conducting paints
- Crack propagation monitoring by plastic films
- Crack detection by penetrating paints

Sources of the acoustic emission can be as follows:

- Initiation and development of microdefects.
- Phase (crystalic) transformations in material.
- Plastic deformation related to the initiation of slip bands or to an intensive dislocation movement.
- Ruptures of reinforcing fibres in composite materials, as well as separation between fibres and matrix, delamination, etc.

An existing fracture can be analyzed by methods of fractography.

Detection of body movement

The measured quantities can be of the following types:

- **displacement (change in position**)
- **velocity**
- **acceleration.**

These quantities can be easily mutually recalculated.

With respect to the reference system, the displacements can be

- – displacements of the body as a whole (**rigid body movement**),
- – displacements caused by body **deformation** (distortion).

Displacements caused by the body deformation use to be periodical (**vibrations**).

Basic types of **sensing elements for movement detection**

- **Piezoelectric sensing elements detect the acceleration**.

  They are based on the piezoelectric properties of some crystals capable to induce an electric charge proportional to the load of the crystal. The load – force acting upon the crystal – is proportional to the acceleration (according to Newton's law F = m.a), so that the voltage on the crystal is proportional to the acceleration. These elements are suitable for detection of **fast dynamic processes**.

- **Inductivity sensing elements detect the position or velocity**.
  - Inductivity (the electric resistance of the coil under alternating voltage) can be changed in consequence of changes of the airspace width in a magnetic circuit or by sliding of the core into the coil (it detects changes of **position**).
  - A coil movement in a magnetic field induces a voltage (proportional to the velocity component perpendicular to the magnetic flux lines) in the coil – the **velocity of movement** is detected..

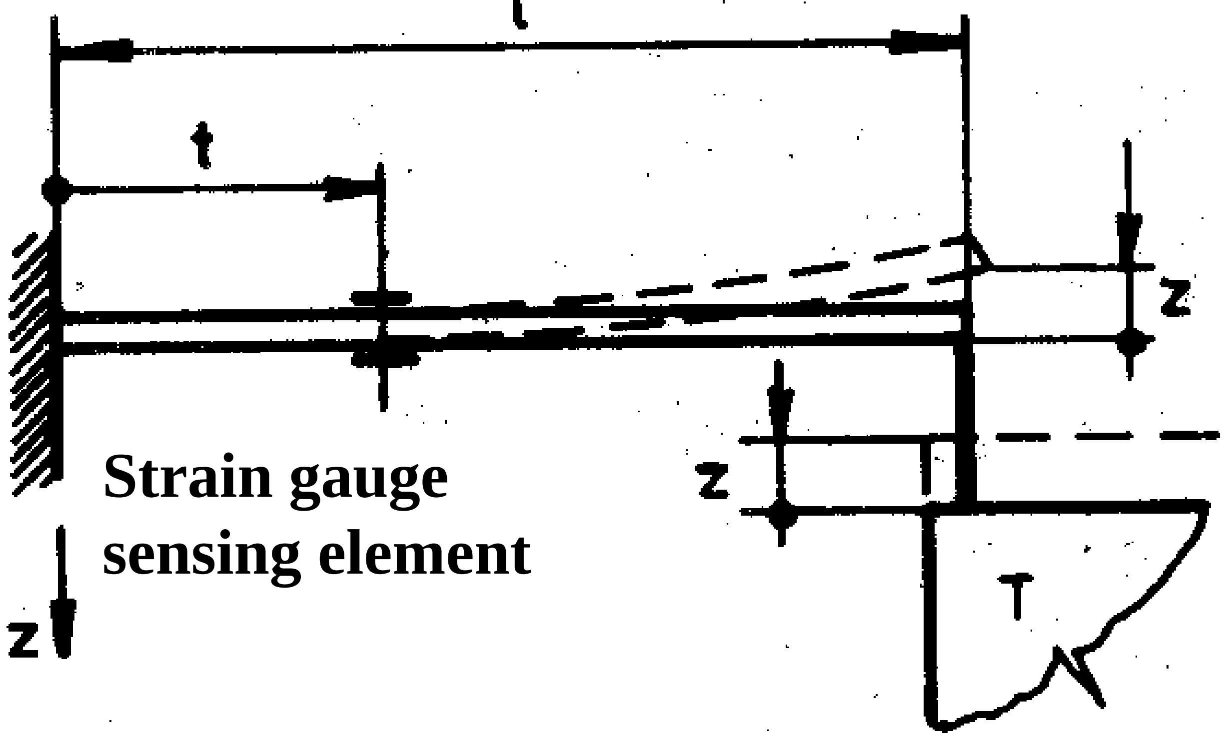
  Inductivity **sensing elements** have a higher inertial mass, therefore they are **not suitable** for detection of **fast dynamic processes**.

- **Capacitor sensing elements detect the change of position**.

  They are based on a change of the capacity caused by either the change in the distance of capacitor plates (electrodes) or by change in their overlap which is applied mostly in rotational arrangement for detection of angles. They are capable to detect **very fast changes** and, as contactless sensors, they are suitable for detection of vibrations of rotating or quickly moving bodies.

- **Strain gauge sensing elements** are based on detection of deformation of a body, mostly a beam or membrane, using electric resistance metalic or semiconductor strain gauges. The measured strain can be recalculated into displacements of the substrate (beam or membrane) by using the corresponding analytical theories.

Position or force sensing element



It holds for the loading force *F* and deflection *z* of the above beam with rectangular cross section (with thickness h):

$$F = \frac{\varepsilon E W_0}{l - t}$$

$$z = \frac{2\varepsilon l^3}{3(l - t)h}$$

Sensing elements for measuring of loads

Loads to be measured can be represented by:

- forces,
- moments (couples),
- pressure.

Principles of sensing elements:

- **strain gauges** – the required load magnitude is calculated using the linear theory of elasticity from the strain value measured in the defined location on the sensing element surface by an electric resistance strain gauge,
- piezoelectric,
- inductivity,
- capacitor.