PPII – equations being at students' disposal for the exam

Any axisymmetric body

$$\varepsilon_r = \frac{du}{dr} \qquad \varepsilon_t = \frac{u}{r} \qquad \varepsilon_z = \frac{dw}{dz} \qquad u = r\varepsilon_t = \frac{r}{E}\left[\sigma_t - \mu(\sigma_r + \sigma_z)\right]$$

Cylindrical vessel

$$\sigma_r = A - \frac{B}{r^2} \qquad \sigma_t = A + \frac{B}{r^2}$$

Rotating disc

$$\sigma_r = A - \frac{B}{r^2} - \frac{3+\mu}{8}\rho\omega^2 r^2 \qquad \sigma_t = A + \frac{B}{r^2} - \frac{1+3\mu}{8}\rho\omega^2 r^2$$

Axisymmetric plate

$$\frac{d^2\upsilon}{dr^2} + \frac{1}{r}\frac{d\upsilon}{dr} - \frac{1}{r^2}\upsilon = -\frac{t(r)}{B} \qquad \approx \qquad \frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(r\upsilon)\right] = -\frac{t(r)}{B}$$

$$\upsilon(r) = c_1 r + \frac{c_2}{r} + \upsilon_p \qquad w(r) = c_1\frac{r^2}{2} + c_2 \ln r + w_p + c_3$$

$$u = -z\upsilon \qquad \upsilon = tg\,\upsilon = \frac{dw}{dr}$$

$$m_r = -B\left[\frac{d\upsilon}{dr} + \mu\frac{\upsilon}{r}\right] \qquad m_t = -B\left[\frac{\upsilon}{r} + \mu\frac{d\upsilon}{dr}\right] \qquad B = \frac{Eh^3}{12(1-\mu^2)}$$

$$\sigma_r = \frac{12m_r}{h^3}z \qquad \sigma_{r\max} = \pm 6m_r / h^2$$

$$\sigma_t = \frac{12m_t}{h^3}z \qquad \sigma_{t\max} = \pm 6m_t / h^2$$

Membrane shell

$$\frac{\sigma_t}{r_t} + \frac{\sigma_m}{r_m} = \frac{p}{h} \qquad 2\pi r h \sigma_m \sin\varphi_m - F_z + F_{rz} = 0$$

Momentum shell

$$u = e^{-\beta z}\left(c_1 \sin\beta z + c_2 \cos\beta z\right) + e^{\beta z}\left(c_3 \sin\beta z + c_4 \cos\beta z\right) + u_{part}$$

$$u_{part} = \frac{r^2}{Eh}\left[p \;\; - \frac{\mu}{r} n_z\right] \qquad \beta = \sqrt[4]{\frac{3\left(1-\mu^2\right)}{r^2 h^2}} \qquad l_0 = 4/\beta \cong 3\sqrt{rh}$$

$$n_z = \frac{Eh}{1-\mu^2}\left[\frac{dw}{dz} + \mu\frac{u}{r}\right] \qquad n_t = \frac{Eh}{1-\mu^2}\left[\frac{u}{r} + \mu\frac{dw}{dz}\right] \qquad \upsilon = \frac{du}{dz}$$

$$m_z = -B\frac{d^2u}{dz^2} \qquad m_t = -\mu B\frac{d^2u}{dz^2} = \mu m_z \qquad t = -B\frac{d^3u}{dz^3}$$

$$B = \frac{Eh^3}{12\left(1-\mu^2\right)} \qquad n_t = \mu n_z + Eh\frac{u}{r}$$

$$\sigma_{t\max} = \frac{n_t}{h} \pm \frac{6m_t}{h^2} \qquad \sigma_{z\max} = \frac{n_z}{h} \pm \frac{6m_z}{h^2}$$

Factor of safety for cyclic loading

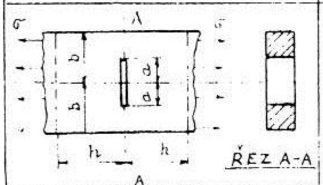
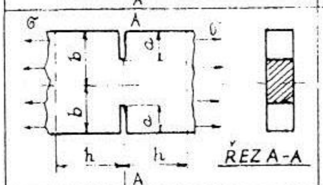
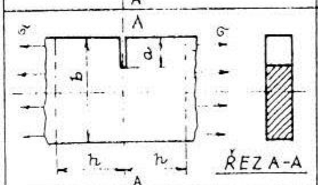
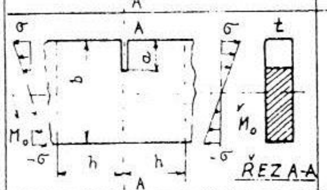
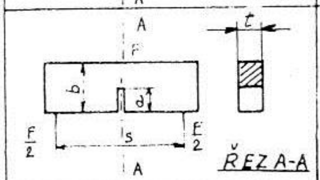
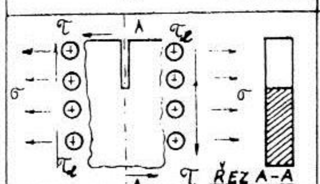
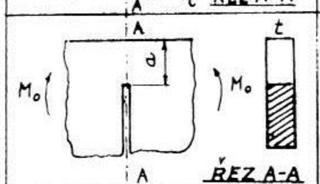
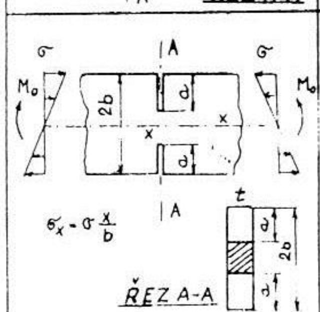
$$k_C = \frac{\sigma_C^*}{\dfrac{\sigma_C^*}{\sigma_C}\psi_\sigma\sigma_m + \sigma_a} \cong \frac{\sigma_C^*}{\sigma_{ae}} \qquad \sigma_C^* = \sigma_C\frac{v\eta}{\beta} \qquad k_C = \frac{k_{C\sigma}\;.\;k_{C\tau}}{\sqrt{k_{C\sigma}^2 + k_{C\tau}^2}}$$

Fracture mechanics (LEFM)

$$\sigma_{ij} = \frac{1}{\sqrt{2\pi r}}\left[K_I \,.\, f_{ij}^{I}(\varphi) + K_{II} \,.\, f_{ij}^{II}(\varphi) + K_{III} \,.\, f_{ij}^{III}(\varphi)\right]$$

$$K_I = \sigma\sqrt{\pi a}\,.\,f\left(\frac{a}{b}\right) \qquad r_k = \frac{\alpha}{\pi}\left(\frac{K_I}{R_e}\right)^2$$

$$a_c \cong \left(\frac{K_{IC}}{\sigma}\right)^2 . \frac{1}{\pi} \qquad v_T = \frac{da}{dN} = c\left(\Delta K_I\right)^m \qquad \Delta N = \frac{2\left(a_f^{1-\frac{m}{2}} - a_0^{1-\frac{m}{2}}\right)}{c\Delta\sigma^m \pi^{\frac{m}{2}}(2-m)}$$

SCHEMA	VZTAH PRO K	S PŘESNOSTÍ PLATÍ PRO
	$K_I = \sigma\sqrt{\pi a}\,\frac{1-0{,}5(\frac{a}{b})+0{,}37(\frac{a}{b})^2-0{,}044(\frac{a}{b})^3}{\sqrt{1-\frac{a}{b}}}$ $K_{II}=K_{III}=0$	0,3% PRO JAKÉKOLIV $\frac{a}{b}$ $a<b$, $\frac{h}{b}\geq 3$
	$K_I = \sigma\sqrt{\pi a}\,\frac{1{,}122-0{,}561(\frac{a}{b})-0{,}205(\frac{a}{b})^2+0{,}471(\frac{a}{b})^3-0{,}19(\frac{a}{b})^4}{\sqrt{1-\frac{a}{b}}}$ $K_{II}=K_{III}=0$	0,5 % PRO JAKÉKOLIV $\frac{a}{b}$ $a<b$, $\frac{h}{b}\geq 2{,}75$
	$K_I = \sigma\sqrt{\pi a}\left[1{,}12-0{,}231(\frac{a}{b})+10{,}55(\frac{a}{b})^2-21{,}72(\frac{a}{b})^3+30{,}39(\frac{a}{b})^4\right]$ $K_{II}=K_{III}=0$	0,5% PRO $\frac{a}{b}\leq 0{,}6$ $a<b$, $\frac{h}{b}\geq 1$
	$\sigma=\frac{6M_o}{tb^2}$ $K_I = \sigma\sqrt{\pi a}\left[1{,}122-1{,}4(\frac{a}{b})+7{,}33(\frac{a}{b})^2-13{,}08(\frac{a}{b})^3+14{,}0(\frac{a}{b})^4\right]$ $K_{II}=K_{III}=0$	0,2 % PRO $\frac{a}{b}\leq 0{,}5$ $a<b$, $\frac{h}{b}\geq 2$
	$\sigma=\frac{6M_o}{tb^2}\quad (M_o=\frac{Fs}{4})$ $K_I = \sigma\sqrt{\pi a}\left[1{,}107-2{,}12(\frac{a}{b})+7{,}71(\frac{a}{b})^2-13{,}55(\frac{a}{b})^3+14{,}25(\frac{a}{b})^4\right]$ $K_{II}=K_{III}=0$	0,2 % PRO $\frac{a}{b}\leq 0{,}6$ $a<b$, $\frac{s}{b}=8$
	$K_I = 1{,}1215\,\sigma\sqrt{\pi a}$ $K_{II} = 1{,}1215\,\tau\sqrt{\pi a}$ $K_{III} = \tau_l\sqrt{\pi a}$	K_I, K_{II} PRAKTICKY PŘESNÉ K_{III} PŘESNÉ
	$K_I = 3{,}975\,\frac{M_o}{ta\sqrt{a}}$ $K_{II}=K_{III}=0$	DO 0,1%
	$\sigma=\frac{3M_o}{2tb^2}\qquad K_{II}=K_{III}=0$ $\sigma_N=\frac{3M_o}{t2(b-a)^2}=\frac{\sigma}{1-(\frac{a}{b})^2}$ $K_I=\sigma\sqrt{\pi a}\cdot f_1(\frac{a}{b})=\sigma_N\sqrt{\pi a}\,f_2(\frac{a}{b})$ $f_1(\frac{a}{b})=\frac{\frac{4}{3\pi}\{1+\frac{1}{2}(\frac{a}{b})+\frac{3}{8}(\frac{a}{b})^2+\frac{5}{16}(\frac{a}{b})^3\}-0{,}47(\frac{a}{b})^4+0{,}663(\frac{a}{b})^5}{\sqrt{(1-\frac{a}{b})^3}}$ $f_2(\frac{a}{b})=(1-\frac{a}{b})^2\,F_1(\frac{a}{b})$	LEPŠÍ NEŽ 1% $a<b$