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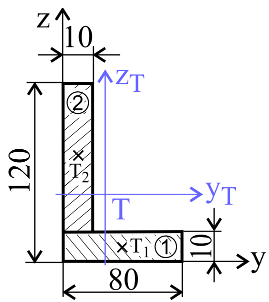
principal c.s.

properties

The principal central axes are characterized, in addition to their point of intersection in the centre of gravity, by the zero deviatoric quadratic moment related to them. We can turn profit of basic properties of the quadratic moments and to divide the section into two rectangular parts (1 and 2) with known values of the quadratic moments. There are more possible successions of steps, we choose one of them:

1. We choose a global coordinate system $(0, y, z)$.
2. We divide the section into two rectangular parts (1 and 2) with known values of the quadratic moments.
3. We calculate the coordinates of the centre of gravity in the global coordinate system (y_T, z_T).
4. We transform the known formulas for quadratic moment of the rectangles to obtain the quadratic moments related to the central axes (T, y_T, z_T), parallel to the global coordinate system of the section (using Steiner's theorem) and then we can summarize both of them.
5. We calculate the position angle of the principal coordinate system (using the condition of the zero deviatoric quadratic moment) and substitute its value into the formulas for transformation of moments by rotation; in this way we obtain the principal central quadratic moments.
6. We check the results using Mohr's representation.

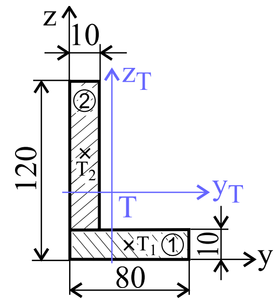


basic shapes

α_h

transformation c.s.

1) Centre of gravity:



centre of gravity

$$y_T = \frac{\sum y_i S_i}{\sum S_i} = \frac{40 \cdot 80 \cdot 10 + 5 \cdot 110 \cdot 10}{80 \cdot 10 + 10 \cdot 110} = 19,7\,\mathrm{mm}$$

$$z_T = \frac{\sum z_i S_i}{\sum S_i} = \frac{5 \cdot 80 \cdot 10 + 65 \cdot 110 \cdot 10}{80 \cdot 10 + 10 \cdot 110} = 39,7\,\mathrm{mm}$$

2) Central quadratic moments:

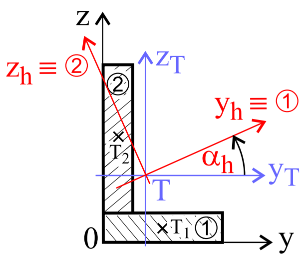
Steiner's theorem

$$J_{y_T} = \frac{80 \cdot 10^3}{12} + (39,7-5)^2 \cdot 80 \cdot 10 + \frac{10 \cdot 110^3}{12} + (39,7-65)^2 \cdot 10 \cdot 110 = 2\;783,2 \cdot 10^3\,\mathrm{mm}^4$$

$$J_{z_T} = \frac{10 \cdot 80^3}{12} + (19,7-40)^2 \cdot 80 \cdot 10 + \frac{110 \cdot 10^3}{12} + (19,7-5)^2 \cdot 10 \cdot 110 = 1\;003,2 \cdot 10^3\,\mathrm{mm}^4$$

3) Calculation of the position angle of the principal coordinate system α_h

$$\alpha_h = \frac{1}{2}\mathrm{arctg}\left(\frac{-2J_{y_T z_T}}{J_{y_T} - J_{z_T}}\right) = 23,8^\circ$$



4) Calculation of the principal central quadratic moments J_{y_h}, J_{z_h}:

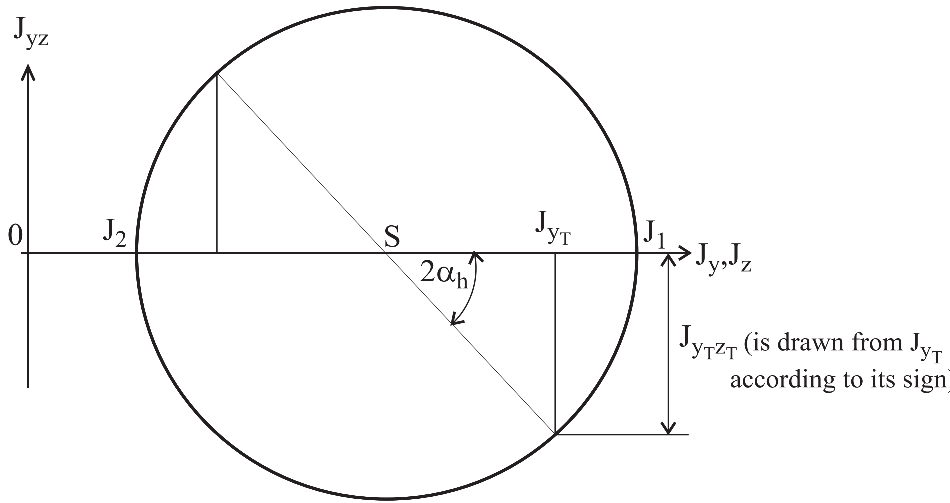
$$J_{y_h} = J_{y_T}\cos^2\alpha_h - J_{y_T z_T}\sin 2\alpha_h + J_{z_T}\sin^2\alpha_h = 3\,211,6 \cdot 10^3\,\mathrm{mm}^4 = J_1$$

$$J_{z_h} = J_{z_T}\cos^2\alpha_h + J_{y_T z_T}\sin 2\alpha_h + J_{y_T}\sin^2\alpha_h = 574,8 \cdot 10^3\,\mathrm{mm}^4 = J_2$$

5) Graphical evaluation of the principal coordinate system angle and of the principal quadratic moments:

Since the coordinate system (T, y_T, z_T) is central, the axes we are seeking for create the principal central coordinate system.

Scale factor: m_J : $\quad 40 \cdot 10^3\,\mathrm{mm}^4 \cong= 1\,\mathrm{mm}$



$$J_1 = l_{J_1} \cdot m_J = 80,3 \cdot 40 \cdot 10^3 = 3\ 212 \cdot 10^3\,\mathrm{mm}^4$$

$$J_2 = l_{J_2} \cdot m_J = 14,4 \cdot 40 \cdot 10^3 = 576 \cdot 10^3\,\mathrm{mm}^4$$

$$2\alpha_h = 46,5^\circ \quad \Rightarrow \quad \alpha_h = 23,3^\circ$$