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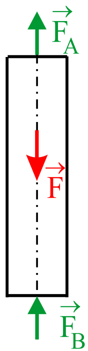
limit state of elasticity
supported bar
deformation loading
supports
statical analysis
conditions nonhomogeneou

Analysis:

Objectives: check of the limit state of elasticity.

Classification of the bar: straight bar, supported, loaded by forces and deformations (the bar is produced with a **small** inaccuracy, it is the value of δ longer than the distance between the plates of the base, to which it should be welded by both of its ends; after assemblage, the bar is thus loaded also by deformation).

Free body diagram:



Resultants consisting of three force and three moment components can be created in each of the (fixed in 3D space) supports A and B. If we ensure a sufficient coaxiality and alignment of the supports, the components without any significant external loads in their direction will be negligible, and the only one significant component of the support resultant will be the force parallel to the bar centreline and thus also to the external loads.

Statical analysis: $\nu = 1, \qquad \mu = 2$ (a system of forces acting in the same line)
$s = \mu - \nu = 1 \quad \Rightarrow$ statically onefold indeterminate system

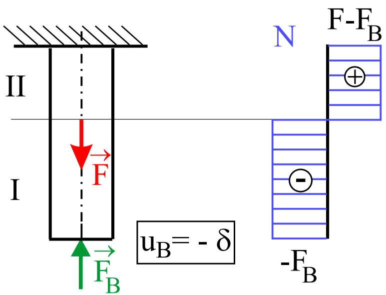
Applicable conditions of static equilibrium: $F_x: \quad F_A - F + F_B = 0$

Released structure:

Distribution of the inner resultant:

$x \in (0, b): \quad N_I = -F_B$

$x \in (0, a): \quad N_{II} = F - F_B$



released structure

Modification of the compatibility equations:

$$u_B = \frac{-F_B}{ES} b + \frac{F - F_B}{ES} a = -\delta$$

The *negative* sign in the right-hand side of the equation expresses that the elongation created by internal forces must be negative; if the bar is namely the δ value longer, it must be shortened to achieve its nominal length.

displacement

Set of equations:

$$F_A - F + F_B = 0$$

$$\frac{-F_B}{ES} b + \frac{F - F_B}{ES} a = -\delta$$

Solution:

$$F_B = \frac{\delta ES + Fa}{a + b}, \qquad F_A = F + F_B = \frac{Fb - \delta ES}{a + b}.$$

Discussion:

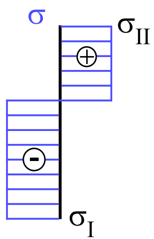
If a state can occur in operation of a statically indeterminate supported body (e.g. in relation to the assemblage of the bars), in which the bar is loaded only by deformation, this state should be checked as well (by substitution $F = 0$ into these relations). The stress state can be worse in some points of the bar under these conditions than when the force is acting.

a)) state after assemblage:
$F = 0 \quad \Rightarrow \quad F_B = \dfrac{\delta ES}{a+b}$ - all the bar is loaded in compression.

b) state after assemblage and loaded by the force $\vec{F}$:
$F_B = \dfrac{\delta ES + Fa}{a+b} \quad \Rightarrow$ Tension or compression can occur in the interval II in dependence on the values δES and Fa.

Distribution of stresses:

stress



Since the bar is loaded in simple tension (compression), the stress is constant across the section and can be calculated using the following formula:

$$\sigma(x) = \frac{N(x)}{S(x)} : \qquad \sigma_I = \frac{N_I}{S} = \frac{F_B}{S}, \quad \sigma_{II} = \frac{N_{II}}{S} = \frac{F_A}{S}$$

Check for the limit state of elasticity:

limit state

We determine the section with extreme stress (dangerous section) from the stress distribution and calculate the safety factor against the limit state of elasticity:

$$k_K = \frac{\sigma_K}{\sigma_{\max}}.$$

Note: The stress states in supports A and B depend, in addition to the notch effect, on the technology of welding. Therefore we do not check them, although just these points can be dangerous. Similar, a stress concentration occurs in the point of action of the force $\vec{F}$, but this concentration is not specified in the example.