Back to problem

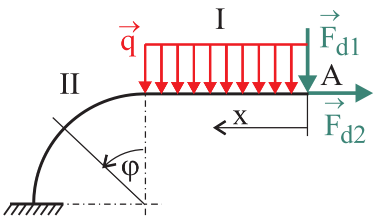
Analysis:

curved bar

Classification of the bar: in-plane problem, a curved bar, supported in the statically determinate way, loaded by external (distributed) loads. As we are about to solve (using Castigliano's theorem) the displacement components (horizontal and vertical) in a point where no concentrated force is acting, we introduce the additional forces $\vec{F}_{d1} = 0$ in horizontal and $\vec{F}_{d2} = 0$ in vertical directions.

Castigliano's theorem

integral approach



The bar is loaded by combination of tension, shear and flexion. It can be derived that the bending moment is the only significant component of inner resultants, if the length of the bar is at least one order higher than the dimensions of the cross section. Therefore we solve the distribution of only this one component of the inner resultants.

$$M_o^I(x) = -F_{d1}x - \frac{qx^2}{2}$$

Example 627

$$M_o^{II}(\varphi) = -F_{d1}(a + r\sin\varphi) - F_{d2}r(1-\cos\varphi) - qa\left(\frac{a}{2} + r\sin\varphi\right)$$

Horizontal v_A and vertical w_A component of the displacement:

$$
\begin{aligned}
v_A &= \frac{\partial W}{\partial F_{d1}} = \int\limits_{\gamma} \frac{M_o}{EJ} \cdot \frac{\partial M_o}{\partial F_{d1}} \mathrm{d}s = \\
&= \frac{1}{EJ} \left\{ \int\limits_0^a -\frac{qx^2}{2} \cdot (-x)\mathrm{d}x + \int\limits_0^{\pi/2} -qa\left(\frac{a}{2} + r\sin\varphi\right) \cdot [-(a + r\sin\varphi)] \cdot r\mathrm{d}\varphi \right\} = \\
&= \frac{1}{EJ} \left\{ \frac{qa^4}{8} + qar\left[\frac{\pi a^2}{4} - \frac{3}{2}ar[\cos\varphi]_0^{\pi/2} + r^2[\frac{\varphi}{2} - \frac{1}{4}\sin 2\varphi]_0^{\pi/2}\right]\right\} = 0,063\,\mathrm{m} \\
w_A &= \frac{\partial W}{\partial F_{d2}} = \frac{1}{EJ} \left\{ \int\limits_0^a 0\mathrm{d}x + \int\limits_0^{\pi/2} -qa\left(\frac{a}{2} + r\sin\varphi\right) \cdot [-r(1 - \cos\varphi)] \cdot r\mathrm{d}\varphi \right\} = 0,0085\,\mathrm{m}
\end{aligned}
$$

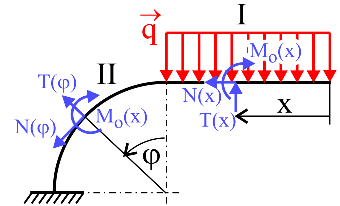
Check for the linearity of the problem: linearity

The necessary conditions of the linearity of the problem are

a) the material is linear elastic - valid up to the yield stress,
b) the displacements are small in comparison with the dimensions of the bar, the criterion of linearity in bending is that the slopes are small (less than 0,1 rad)
c) strains are small (less than about 1%, always satisfied at steels under the yield stress value)
d) boundary conditions are linear (the direction of the load does not change during the loading process) - satisfied without any further conditions if the slopes are small.

1. Distribution of inner resultants:

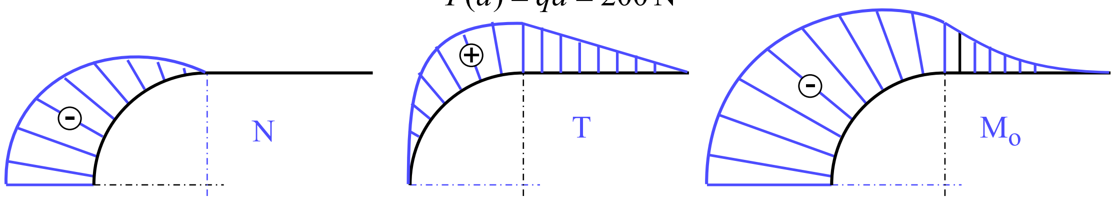
   I. interval: $x \in (0; a)$ II. interval: $x \in (0; \pi/2)$



$$
\begin{aligned}
N^I(x) &= 0, & N^{II}(\varphi) &= -qa\sin\varphi, \\
T^I(x) &= qx, & T^{II}(\varphi) &= qa\cos\varphi, \\
M_o^I(x) &= -\frac{qx^2}{2}, & M_o^{II}(\varphi) &= -qa\left(\frac{a}{2} + r\sin\varphi\right)
\end{aligned}
$$

2. Dangerous section will be in the location with maximum bending moment, namely in location of the fixed support.

$T(a) = qa = 200\,\text{N}$

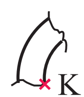


$N(\pi/2) = qa = 200\,\text{N}$ $M_o(\pi/2) = -400\,\text{Nm}$

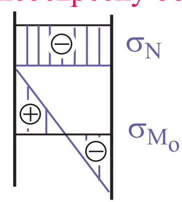
3. The dangerous point in this section

   can be evaluated from the stress distribution across the section.

nebezpečný průřez:
vetknutí



nebezpečný bod K



σ_N

σ_{M_o}

4. We compare the extreme stress with the yield stress

$$\begin{aligned}\sigma_{max} &= \frac{4N(\pi/2)}{\pi d^2} + \frac{32M_o(\pi/2)}{\pi d^3} = \frac{4 \cdot 200}{0,04^2\pi} + \frac{32 \cdot 400}{0,04^3 2\pi} = \\ &= 0,159 \cdot 10^6 + 63,66 \cdot 10^6 = 63,8 \cdot 10^6\,\mathrm{Pa} = 63,8\,\mathrm{MPa} < \sigma_K\end{aligned}$$