Back to problem

Analysis:

A system of three straight slender bars loaded in their joint, each of the bars supported by the base, all the supports modelled as pin supports. The bar No. 2 is produced with a **small** production inaccuracy (the value of shorter) so that the structure is loaded also by deformation.

system of bars

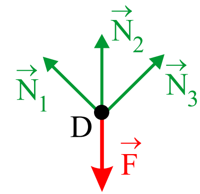
deformation loading

analysis

released structure

Free body diagram:

We isolate the joint D as a free body:

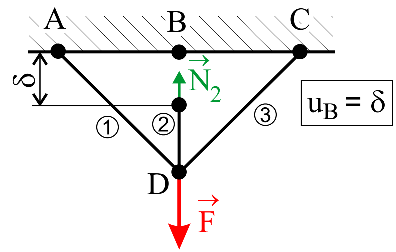


Statical analysis: $\mu = 3$, $\nu = 2$ (central in-plane system of forces (intersecting each other in one point))
$s = \mu - \nu = 0 \quad \Rightarrow$ statically onefold indeterminate problem

Released structure:

The easiest way of how to create the primary (released) structure is usually to release a joint between one of the bars and the base; the deformation of the base is negligible so that the compatibillity equation has a simpler initial form.

For instance, we release the bar No. 2 in the point B and introduce the force N_2 (statical redundant) here; the corresponding compatibility equation is $u_B = \delta$.
We express the displacement of the point B using Castigliano's theorem so that the positive orientation of displacement will be identical with the positive orientation of the force N_2 and thus also with the displacement needed during assemblage (the positive value of δ in the right-hand side of the compatibility equation):



Castigliano's theorem

$$u_B = \frac{\partial W}{\partial N_2} = \sum_{i=1}^{3} \frac{N_i l_i}{ES} \frac{\partial N_i}{\partial N_2} = \delta$$

Applicable conditions of static equilibrium:

static equilibrium

$\sum F_x = 0 : \quad -N_1 \sin\alpha + N_3 \sin\alpha = 0$
$\sum F_y = 0 : \quad N_1 \cos\alpha + N_2 + N_3 \cos\alpha - F = 0$
We express the **forces in the bars** using the statical redundant:

applicable conditions

$$N_1 = N_3, \qquad N_1 = \frac{F - N_2}{2\cos\alpha}.$$

Modification of the compatibility equation and calculation of the forces in the bars:

$$u_B = \frac{1}{ES}\left[\frac{(F - N_2) l_1}{2\cos\alpha}\left(-\frac{1}{2\cos\alpha}\right) \cdot 2 + N_2 l_2 \cdot 1\right] = \frac{1}{ES}\left[\frac{(N_2 - F) l}{2\cos^3\alpha} + N_2 l\right] = \delta$$

$$N_2 = \frac{Fl + 2\delta ES\cos^3\alpha}{l(2\cos^3\alpha + 1)}, \qquad N_1 = N_3 = \frac{F - N_2}{2\cos\alpha}$$

Distribution of stresses: stress

Since each of the bars (of the same cross section) is loaded in simple tension (compression) by the normal force constant along the bar centreline, the stress is constant in each of them and can be calculated using the following formula:

$$\sigma(x) = \frac{N(x)}{S(x)}: \qquad \sigma_1 = \sigma_3 = \frac{N_1}{S}, \quad \sigma_2 = \frac{N_2}{S}.$$

Check for the limit state of elasticity: limit state

Since the stress is constant in each of the bars and they are made of the same material (with the same yield stress), we can calculate the safety factor against the limit state of elasticity valid for all the system by substituting the maximum of the calculated stress values into the formula:

$$k_K = \frac{\sigma_K}{\sigma_{\max}}.$$

Note: the calculation does not account for the technology of the joints that is decissive for the choice of the acceptable safety factor value.