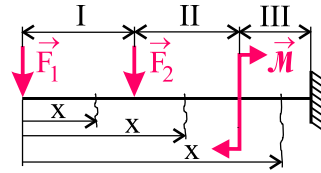


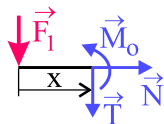
*Classification of the bar:* straight, loaded by external forces, supported.  
*Statical analysis:*  $\mu = 3$ ,  $\nu = 3$  (a general in-plane system of forces)  
 $s = \mu - \nu = 3 - 3 = 0$

Back to  
 problem  
 analysis

Now we usually isolate the bar as a free body and calculate the reactions in supports. The bar in question, however, has a free end, so that we need not to calculate the reactions; we can isolate elements containing this free end. We divide the bar into three intervals, namely in the points where external forces or couples are acting on the bar.



# Integral approach:



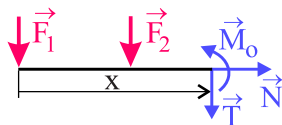
**Inner resultants in the interval I:**  $x \in (0; a)$

$$F_x: N(x) = 0,$$

$$F_z: T(x) = -F_1$$

$$M_y: M_o(x) = -F_1 x$$

integral  
approach

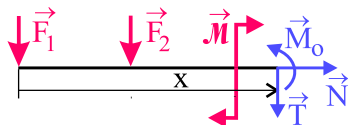


**Inner resultants in the interval II:**  $x \in (a; a + b)$

$$F_x: N(x) = 0,$$

$$F_z: T(x) = -F_1 - F_2,$$

$$M_y: M_o(x) = -F_1 x - F_2(x - a),$$



**Inner resultants in the interval III:**  $x \in (a + b; l)$

$$F_x: N(x) = 0,$$

$$F_z: T(x) = -F_1 - F_2,$$

$$M_y: M_o(x) = -F_1 x - F_2(x - a) + \mathcal{M}.$$

## Differential approach:

differential  
approach

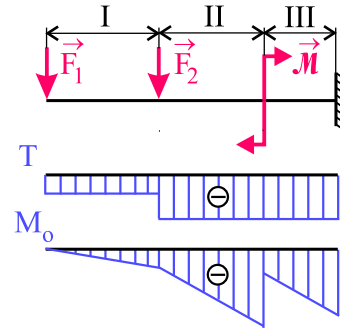
## Graphical representation of the distribution of components of inner resultants:

### Shear force:

- I. interval: constant shear force  $T(x) = -F_1$
- II. interval: shear force  $F_2$  increased  $\rightarrow T(x) = -(F_1 + F_2)$
- III. interval: without changes in forces  $\rightarrow T(x) = -(F_1 + F_2)$

There is no distributed load in all the intervals  $q_T(x) = 0$  :

$$\frac{dT(x)}{dx} = -q_T(x) = 0 \rightarrow T(x) \text{ parallel to the } x \text{ axis.}$$



### Bending moment:

rules

- I. interval:  $T(x) = \text{const.} \rightarrow \frac{dM_o(x)}{dx} = T(x) = \text{const.} \rightarrow$  linear function of  $M_o(x)$ .  
 $T(x) < 0 \rightarrow$  function  $M_o(x)$  is decreasing with zero value in the end of the bar, because there is no couple acting upon this end.
- II. interval:  $T(x) = \text{const.} \rightarrow$  function  $M_o(x)$  is linear decreasing again but with a higher slope.
- III. interval: there is a stepwise change in the location of the couple. Shear force is constant and of the same value as in the 2nd interval so that the representation of  $M_o(x)$  is a straight line parallel to that valid for the 2nd interval.