1. Mechanics of materials - introduction

Elasticity theory is one of basic parts of solid mechanics and one of fundamental courses in mechanical engineering studies. When first technical universities were founded in the 19^{th} century, mechanics never missed among the first courses taught there. The need of safe, reliable and failure-free operation of the designed machines became more and more important. This need motivated the development of civil and mechanical engineering and contributed substantially to establishing of mechanics of materials and later of the elasticity theory as a separate scientific branch.

This long tradition causes also some problems, e.g. in translation of traditional terms. In contrast to English, Czech language uses some traditional terms which cannot be translated exactly in English. Solid mechanics, or mechanics of solids, as a branch of mechanics dealing with solids (i.e. bodies), has its unambiguous equivalent in Czech. However, the term used commonly in Czech for that part of solid mechanics which deals with evaluation of stresses and strains and with failure prediction can be translated in English as elasticity and strength theory. The topics of this branch correspond to the term mechanics of materials used frequently in English. Both of these terms are a little bit out of date. The term mechanics of materials originate from the time when dynamics dealt with rigid body motions only. On the other side, elasticity theory describes only one model of material behaviour and up-to-date materials show more and more inelastic properties. Also strength is not a unique characteristic of material related to its failure. Most of failures are caused by material fatigue; it is a complex process which starts by changes at the microstructure level of material, continues with crack initiation and propagation till the fracture of the body. This process is investigated by a special part of mechanics of materials - fracture mechanics, and the only one characteristic value - *strength* or *ultimate stress* - cannot be sufficient for description of such a complex process.

Because of the above reasons, mechanics of materials or strength of materials is the title of this course which deals with two basic topics: **stress and strain analysis and evaluation of failure risk**. As a failure of the body can be caused by various mechanisms (fracture, excessive elastic deformation, plastic deformation, buckling etc.) a general term "limit states" will be used for all the states of the body which are unacceptable from the viewpoint of body operation. Therefore this second part of the mechanics of materials will be called "theory of limit states". The approaches and conventions of this interactive text are based on the teaching text [Janíček, Ondráček, Vrbka, Burša: Mechanika těles. Pružnost a pevnost I].

evaluation of failure risk

1.1. Objectives of the mechanics of materials

The objectives of the mechanics of materials can be defined as a prevention of failures of machines and machinery equipments which are caused by excessive load and deformation, as well as reconstruction of causes of failures which have occurred in operation of the machines before the end of their expected lifetime.

Mechanics of materials helps the designer (mechanical engineer) to determine the shape and dimensions of component parts of machines and structures with respect to the safety, lifetime, economics, eventually with taking also other aspects into account (e.g. esthetics, environmental compatibility, ergonomy, etc.).

The basic task of mechanics of materials can be defined as analysis of the influence of body loads on its stress and strain states with respect to the risk of limit states (occurrence of failures).

The designer's aspiration is to ensure the operational efficiency of the designed equipments, i.e. to minimise the unwanted consequences of contingent limit states or, in opposite, to exploit the consequences of processes related to the limit states in production (e.g. technological processes of moulding, based on plastic deformation or cutting, based on fracture of material). The designer has to formulate and to solve problems concerning mechanics of materials and to exploit his knowledge and experience acquired in solving problems of mechanics.

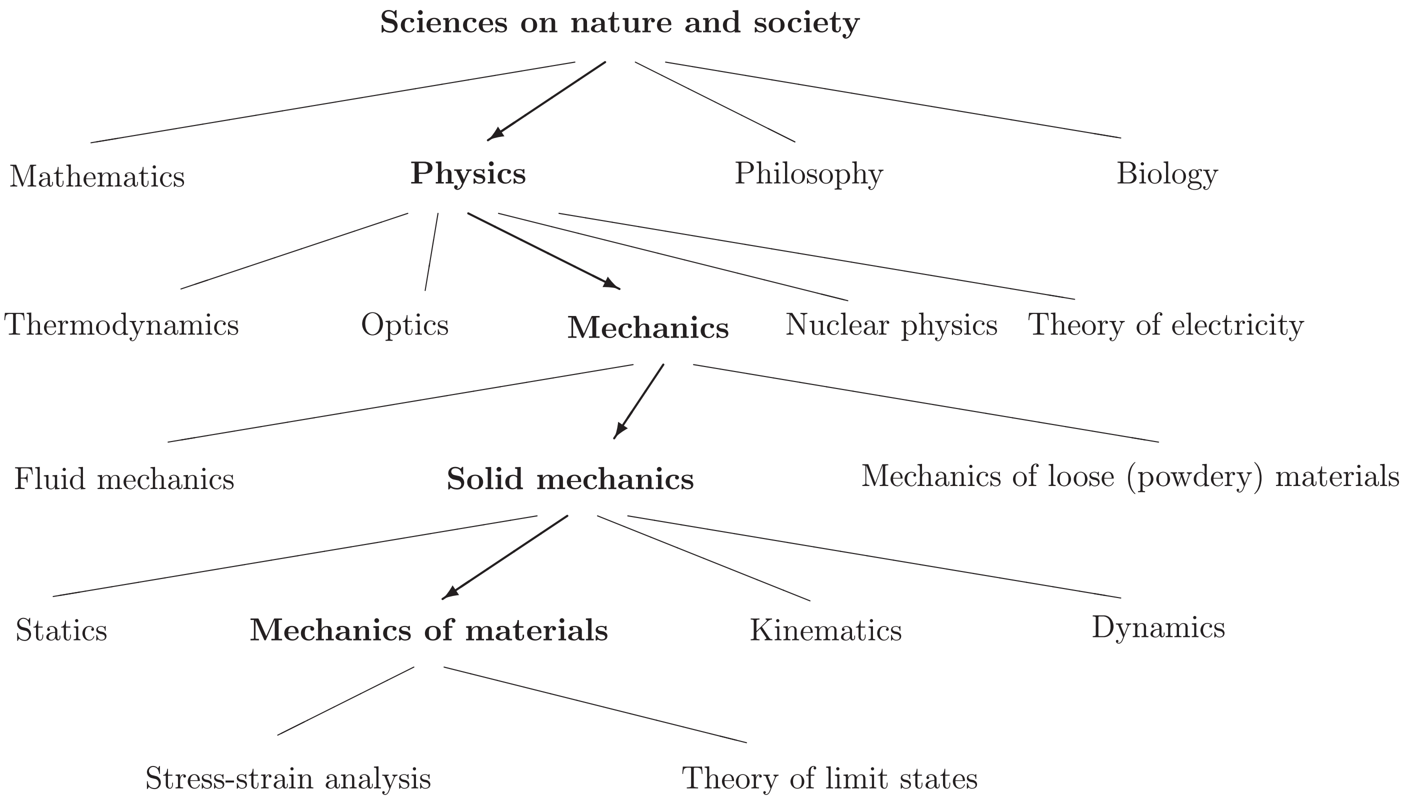
The fundamental difference between an **example** and a **problem** in mechanics (or in general as well) is as follows:

- **example** is formulated by teacher and its solution is rather routine (it exercises the respective computational procedures or verifies that the solver have managed them);
- **the problem** must be sometimes formulated by the solver himself, the solver needs to get the information necessary for the problem solution, to approach the problem in a creative way and to take advantage of the experience acquired by solutions to **examples** and other problems. **It is important to distinguish between direct (forward) and indirect (inverse) problems as well.**

Naturally only **examples** compose the contents of studies in this course and other courses as well, but these **examples** should be accomplished by basic information on practical mechanical problems where the managed procedures can be used.

1.2. Relations between various branches and courses of mechanics

In the following (not complete, naturally) scheme, the position of mechanics of materials among the other scientific branches is illustrated:



1.3. Approaches in mechanics of materials

a) **Intuitive** - the proposal of the way of how to solve the problem is based on the solver's knowledge and experience but the solver is not able to give exact reasons for the correctness of his decisions or for optimum material utilisation. This approach is primary and very important for a designer but it cannot be sufficient. Nothing but intuition can help the designer to choose from the plenty of possible solutions those several ones that promise to be the best. These several solutions, however, must be assessed by other approaches.

b) **Computational** - it is based on creation of a computational model, i.e. on introducing such simplifications which ensure the capability of the designer to solve the computational model in a reasonable defined time space with the hardware being at his disposal and, on the other hand which ensure an acceptable agreement with reality.

   Computational models can be
   - analytical - theory of bars, beams, shells, plates etc.
   - numerical - finite element method, boundary element method etc.

c) **Experimental** - experiment can be carried out using a real object or its material (physical) model. Experiments with the real object are very expensive and time-consuming; in some cases experiments with real objects are quite impossible (nuclear power stations, planes etc.) or they are so expensive that they are carried out only after a comprehensive computational modelling (e.g. crash tests of cars). On the other hand, existence of appropriate measuring methods and equipment for their execution is a necessary condition of experiment with a model, as well as fulfilling of certain criteria which enable the transfer of results between the model and the real technical object (e.g. water turbines). Experiment is unavoidable in any computational modelling; it ensures the necessary input data (e.g. material properties) and the verification of the results and of the model itself.

1.4. Fundamental terms in mechanics of materials

Mechanics of materials as a scientific branch uses a lot of terms; full and unambiguous definition of these terms is unavoidable for understanding of all effects, relations and processes we will deal with. A unified understanding of all these terms is a fundamental assumption of a creative engineering aproach; the up-to-date concept of this approach is based on a continuous communication among designers, technologists, computation specialists and other specialists (environmentalists etc.).

Mechanical motion – it has been defined in the course of statics; statics, however, dealt with a rigid body motion only. The motion has some additional components important especially from the viewpoint of limit states assessment.

Components of mechanical motion:

a) rigid body motion,
b) deformation,
c) splitting - independent motion of parts of the original body.

In some cases it is not easy to distinguish between rigid body motion and deformation (e.g. crash tests of cars). In this course of mechanics of solid materials we will deal only with bodies which do not move with regard to the basic frame (with an only exception of constant rotation speed around a fixed axis). The frame is supposed to be connected with an inertial system, otherwise inertial (virtual) forces had to be added to the real forces acting on the body (this is necessary, e.g., if the chosen basic frame is an accelerating or decelerating car or a vehicle going through a curvilinear trajectory). The approaches of static analyses assume all the forces to be constant in time. The body deformation can indeed induce its failure but we will not deal in greater detail with the failure process itself. deformation

Real and abstract body

Real body represents a basic element of a mechanical system; in the process of solution, it is replaced by a computational model, i.e. by a **theoretical (abstract) body** showing the following properties:

- it is continuous,
- it is continuously deformable up to failure,
- its geometry is defined at the technical resolution level , resolution
- material properties are specified by material characteristics , level
- its accordance with the real body is limited on the **properties substantial** for the problem solved.

The term **substantial properties** of the body can be explained using the following example:

a piston rod of a combustion engine is an example of a real body; the following levels of computational models can be used in solving various mechanical problems concerning this body:

- if we solve the forces acting among the piston rod and the other parts of the engine when the piston is loaded by gas pressure, we need only the well-known information on the straight centerline of the rod and on its (rotational) couplings, and the value of distance between centers of both rotational couplings is the only geometrical information needed.

  Then the computational model can be presented by the following sketch:

- if the inertial forces acting on the piston rod are substantial (high-speed engines), we need to know the complete geometry of the piston rod, its density and kinematic parameters of its motion (vectors of velocity and acceleration in all points of the body),
- if stress-strain analysis is to be realised, then we need to know (in addition to all the above information) some material characteristics of the material of the rod (elasticity and strength parameters of the material used incl. the temperature dependance of these properties and the range of the operational temperatures),
- if a long-term reliability of the piston rod is to be assessed, we need to know all factors influencing the fatigue behaviour: quality of the surface, its technological processing such as hardening, cementation, nitration etc.