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Analysis:

A system of three straight slender bars, each of the bars supported by the base, all the supports modelled as pin supports. The system is loaded by a force acting in the joint and by temperature changes.

system of bars

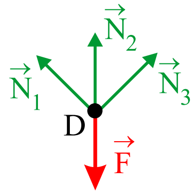
loading

Free body diagram:

We isolate the joint D as a free body:

Statical analysis: $\mu = 3$, $\nu = 2$ (central in-plane system of forces (intersecting each other in one point))

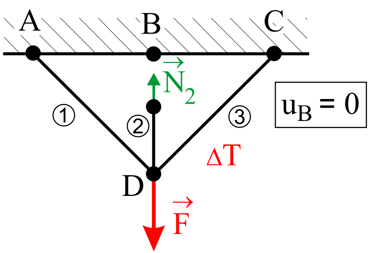
$s = \mu - \nu = 1 \quad \Rightarrow$ statically onefold indeterminate problem



analysis

Released structure:

The easiest way of how to create the primary (released) structure is usually to release a joint between one of the bars and the base; the deformation of the base is negligible so that the compatibillity equation has a simpler initial form. For instance, we release the bar No. 2 in the point B and introduce the following compatibility equation in point B: $u_B = 0$. The displacement in this condition must be created by force as well as by temperature loads.



released structure

Applicable conditions of static equilibrium: $\sum F_x = 0: \quad -N_1 \sin\alpha + N_3 \sin\alpha = 0$
$\sum F_y = 0: \quad N_1 \cos\alpha + N_2 + N_3 \cos\alpha - F = 0$
We express the **forces in the bars** using the statical redundant:

static equilibrium

applicable conditions

$$N_1 = N_3, \qquad N_1 = \frac{F - N_2}{2\cos\alpha}.$$

Modification of the compatibility equation and calculation of the forces in the bars:

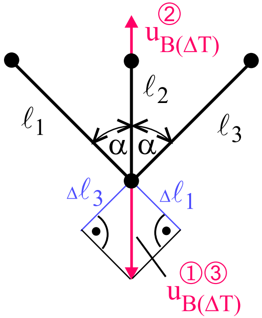
We have to express the compatibility equation using force and temperature loads; there are two possibilities of solving this:

1. the displacement created by external force can be expressed using Castigliano's theorem and that created by temperature load using **geometrical relations**; this approach is enabled by the symmetry of the problem:

Castigliano's theorem

$$\begin{aligned} u_{B(F)} &= \frac{\partial W}{\partial N_2} = \sum_{i=1}^{3} \frac{N_i l_i}{ES} \frac{\partial N_i}{\partial N_2} = \frac{(F - N_2)l}{2E_m S \cos^2\alpha} \left(-\frac{1}{2\cos\alpha}\right) \cdot 2 + \frac{N_2 l}{E_o S} \cdot 1 = \\ &= \frac{(N_2 - F)l}{2E_m S \cos^3\alpha} + \frac{N_2 l}{E_o S} \end{aligned}$$

Moreover, the bars are heated to be the value of ΔT warmer, they will show an additional elongation of $\Delta l_i = l_i \alpha \Delta T$:



$$u^{(1+3)}_{B(\Delta T)} = \frac{\Delta l_1}{\cos\alpha} = \frac{l_1 \alpha_m \Delta T}{\cos\alpha} = \frac{l \alpha_m \Delta T}{\cos^2\alpha}, \quad u^{(2)}_{B(\Delta T)} = l \alpha_o \Delta T$$

$$u_{B(\Delta T)} = -u^{(1+3)}_{B(\Delta T)} + u^{(2)}_{B(\Delta T)} = -\frac{l}{\cos^2\alpha}\alpha_m \Delta T + l\alpha_o \Delta T$$

$$u_B = u_{B(F)} + u_{B(\Delta T)} = \frac{(N_2 - F)l}{2E_m S\cos^3\alpha} + \frac{N_2 l}{E_o S} - \frac{l}{\cos^2\alpha}\alpha_m \Delta T + l\alpha_o\Delta T = 0 \quad \Rightarrow$$

$$N_2 = \frac{2\left(\alpha_m - \alpha_o\cos^2\alpha\right)\Delta T E_m E_o S\cos\alpha + F E_o}{E_o + 2E_m\cos^3\alpha}, \qquad N_1 = N_3 = \frac{F - N_2}{2\cos\alpha}$$

2. We use the relation derived in the example 414 where the strain energy comprehends temperature changes as well. It holds for the strut frame (N_i = const.,E_i = const., S_i = const.) Příklad 414

$$u_B = \frac{\partial W}{\partial N_2} = \sum_{i=1}^{n} \frac{N_i l_i}{E_i S_i}\frac{\partial N_i}{\partial N_2} + \sum_{i=1}^{n}\frac{\partial N_i}{\partial N_2}\alpha_i \Delta T l_i =$$

$$= \frac{F - N_2}{2E_m S\cos\alpha}\frac{l}{\cos\alpha}\left(-\frac{1}{2\cos\alpha}\right)\cdot 2 + \frac{N_2 l}{E_o S}\cdot 1 +$$

$$+\left(-\frac{1}{2\cos\alpha}\right)\alpha_m \Delta T\frac{l}{\cos\alpha} + 1 l\alpha_o\Delta T + \left(-\frac{1}{2\cos\alpha}\right)\alpha_m\Delta T\frac{l}{\cos\alpha} = 0 \quad \Rightarrow$$

$$N_2 = \frac{2\,(\alpha_m - \alpha_o \cos^2\alpha)\,\Delta T E_m E_o S \cos\alpha + F E_o}{E_o + 2E_m \cos^3\alpha}, \qquad N_1 = N_3 = \frac{F - N_2}{2\cos\alpha},$$

what is the same result as that derived by the previous approach.

Distribution of stresses: stress

Since the bars (of the same cross section) are loaded in simple tension (compression), the stress is constant throughout the cross section and can be calculated using the following formula:

$$\sigma(x) = \frac{N(x)}{S(x)}: \qquad \sigma_1 = \sigma_3 = \frac{N_1}{S}, \quad \sigma_2 = \frac{N_2}{S}.$$

Check for the limit state of elasticity: limit state

Since the bars are made of different materials, we have to calculate the safety factor against the limit state of elasticity for all of the bars of the system and the lowest one of them will determine the safety factor of the whole strut frame:

$$k_{K1} = k_{K3} = \frac{\sigma_{Km}}{|\sigma_1|}, \quad k_{K2} = \frac{\sigma_{Ko}}{|\sigma_2|}, \quad k_K = \min\{k_{K1}, k_{K2}, k_{K3}\}.$$