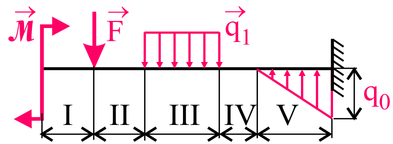
Back to problem analysis

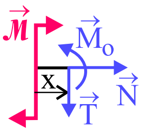
Classification of the bar: straight, loaded by external forces, supported.
Statical analysis: $s = \mu - \nu = 3 - 3 = 0.$

The bar has a free end so that we need not to calculate the reactions; we can isolate elements containing this free end. We divide the bar into five intervals, namely in the points where external forces or couples are acting on the bar or where there is a change in the character of distributed loads.



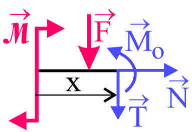
Integral approach:

integral approach



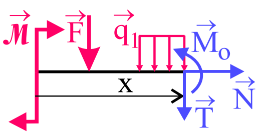
Interval I: $x \in (0; a)$

$$\begin{aligned}
F_x:&\quad N(x) = 0\\
F_z:&\quad T(x) = 0\\
M_y:&\quad M_o(x) = \mathcal{M}
\end{aligned}$$



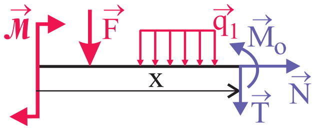
Interval II: $x \in (a; a+b)$

$$\begin{aligned}
F_x:&\quad N(x) = 0\\
F_z:&\quad T(x) = -F\\
M_y:&\quad M_o(x) = \mathcal{M} - F(x-a)
\end{aligned}$$



Interval III: $x \in (a+b; a+b+c)$

$$\begin{aligned}
F_x:&\quad N(x) = 0\\
F_z:&\quad T(x) = -F - q_1(x-a-b)\\
M_y:&\quad M_o(x) = \mathcal{M} - F(x-a) - \frac{q_1(x-a-b)^2}{2}
\end{aligned}$$



Interval IV:

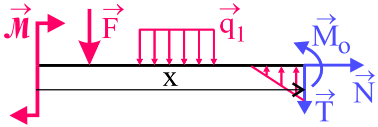
$$x \in (a+b+c; a+b+c+d)$$

$$\begin{aligned}
F_x: &\quad N(x) = 0,\\
F_z: &\quad T(x) = -F - q_1 c\\
M_y: &\quad M_o(x) = \mathcal{M} - F(x-a) - q_1 c(x-a-b-c/2)
\end{aligned}$$

Interval V:

$$x \in (a+b+c+d; a+b+c+d+e)$$

Example 217



$$q(x) = \frac{q_0}{e}(x-a-b-c-d)$$

$$\begin{aligned}
F_x: &\quad N(x) = 0\\
F_z: &\quad T(x) = -F - q_1 c + F_q(x) =\\
&\qquad = -F - q_1 c + \frac{1}{2}\frac{q_0}{e}(x-a-b-c-d)^2\\
M_y: &\quad M_o(x) = \mathcal{M} - F(x-a) - q_1 c(x-a-b-c/2) +\\
&\qquad + \frac{1}{6}\frac{q_0}{e}(x-a-b-c-d)^3
\end{aligned}$$

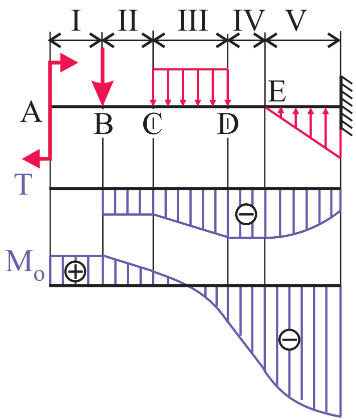
Differential approach:

differential approach

Graphical representation of the distribution of components of inner resultants:

Shear force:

rules



I. interval: zero shear force $T(x) = 0$

II. interval: shear force is the value of F increased, namely $T(x) = -F$,
$\dfrac{\mathrm{d}T(x)}{\mathrm{d}x} = -q_T(x) = 0 \rightarrow T(x)$ parallel to the x axis,

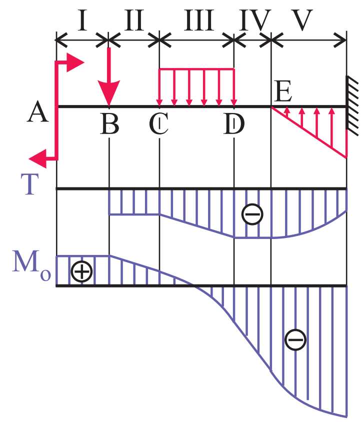
III. interval: $q_T(x) = \text{konst.} > 0 \rightarrow T(x)$ linear decreasing function, $\lim\limits_{x\to C+} q_T(x) \neq \lim\limits_{x\to C-} q_T(x) \quad \rightarrow$ turning point (not a step!) in the C point of the function $T(x)$!

IV. interval: $T(x)$ parallel to the x axis, similar to the interval II

V. interval: $q_T(x) < 0$ (linear) $\rightarrow \quad T(x)$ is a quadratic curve (parabola) with its culmination in the E point,
there is no stepwise change in $q(x)$ in the E point $T(x)$ curve is smooth here, without any turning point, slopes of the tangent lines to the curve are identical on both left-hand and right-hand sides of the E point
$\left(\lim\limits_{x\to E+} q_T(x) = \lim\limits_{x\to E-} q_T(x)\right)$

rules

Bending moment:



I. interval: there is a step in the function $M_o(x)$ in the point A because of the isolated couple acting here, $\frac{\mathrm{d}M_o(x)}{\mathrm{d}x} = T(x) = 0 \rightarrow$ the slope of the tangential line to the function $M_o(x)$ equals zero $\rightarrow M_o(x)$ curve is parallel to the x axis,

II. interval: $\frac{\mathrm{d}M_o(x)}{\mathrm{d}x} = T(x) = \text{const.} < 0 \rightarrow$ function $M_o(x)$ is a decreasing straight line,

III. interval: $T(x) < 0$ linear decreasing, the $M_o(x)$ is a quadratic curve, smoothly joint with the curve valid for the interval II in the point C because there is not a change in $T(x) \rightarrow$ slopes of the tangent lines to the curve are identical on both left-hand and right-hand sides of the C point,

IV. interval: $T(x)$ =const.$\rightarrow$ the function $M_o(x)$ is decreasing again, similar to the interval II but with a higher slope,

V. interval: the function $T(x)$ is quadratic, therefore $M_o(x)$ is a 3rd order curve (cubic parabola), smoothly joint with the curve valid for the interval IV in the point E because there is not a stepwise change in $T(x)$.