Back to problem

Analysis:

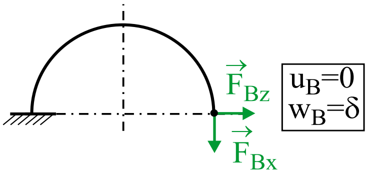
in-plane problem, a curved beam, loaded by deformations.

$\frac{R}{d} = \frac{1000}{50} = 20 \quad \rightarrow \quad$ a beam with low curvature

curved beam

Supports: A fixed support $\xi_A = 3$,
B pin support $\xi_B = 2$

Released structure:



Statical analysis:

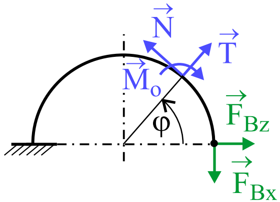
statical analysis

$\mu = 5, \quad \nu = 3$ (general in-plane system of forces)

$s = \mu - \nu = 2 \quad \Rightarrow \quad$ the problem is twofold statically indeterminate

Distribution of inner resultants:

$$\begin{aligned} N(\varphi) &= F_{Bx}\cos\varphi + F_{Bz}\sin\varphi \\ T(\varphi) &= F_{Bx}\sin\varphi - F_{Bz}\cos\varphi \\ M_o(\varphi) &= F_{Bz}R\sin\varphi - F_{Bx}R(1-\cos\varphi) \end{aligned}$$



Modification of the compatibility equation and calculation of the support resultants:

The beam is loaded by combination of tension, shear and flexion. It can be proved that the influence of shear and normal forces on the deflections of the beam is negligible if the length of the beam is substantially higher than the in-plane dimension of the cross section, so that the bending moment $\vec{M}_o$ is the only substantial component of the inner resultants. Example 627

We modify the compatibility equations using Castigliano's theorem: Castigliano's theorem

$$w_B = \frac{\partial W}{\partial F_{Bz}} = \int_0^{\pi} \frac{M_o(\varphi)}{EJ} \frac{\partial M_o(\varphi)}{\partial F_{Bz}} \mathrm{d}s = \int_0^{\pi} \frac{[F_{Bz} R \sin\varphi - F_{Bx} R(1-\cos\varphi)]}{EJ} R \sin\varphi R \mathrm{d}\varphi = \delta$$

$$u_B = \frac{\partial W}{\partial F_{Bx}} = \frac{1}{EJ} \int_0^{\pi} [F_{Bz} R \sin\varphi - F_{Bx} R(1-\cos\varphi)] \, [-R(1-\cos\varphi)] \, R \mathrm{d}\varphi = 0$$

The reactions in the support B: $F_{Bx} = 379$ N, $F_{Bz} = 892$ N.

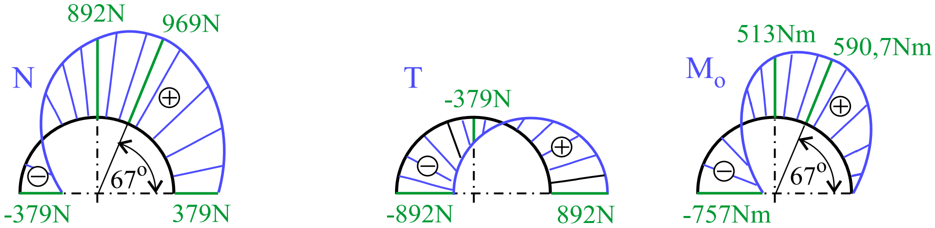
Check for the limit state of elasticity:

limit state

We are about to calculate the safety factor against the limit state of elasticity so that we need to find the dangerous section (i.e. the section with minimum inner resultants or with a stress concentration) and the dangerous point in this section (depending on the character of loading).

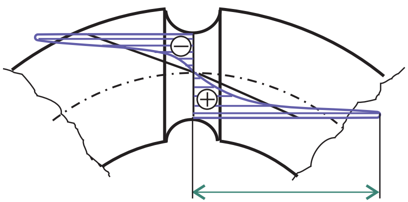
dangerous section

We substitute the calculated support resultant into the equations for distribution of inner resultants and calculate their extreme values:



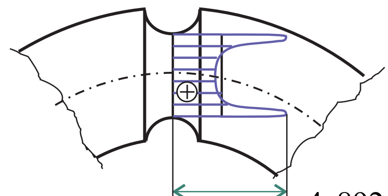
The most significant type of loading is flexion. This statement can be proved by using the formulas for maximum stresses in the notch location for the individual types of loading (tension and flexion). The dangerous section is in the location of the maximum bending moment:

flexion



$$\sigma_{ex}^{Mo} = \alpha\sigma_{nom}^{Mo} = 1,9 \cdot \frac{32 \cdot 513 \cdot 10^3}{\pi \cdot 42^3} = 134 \text{ MPa}$$

tension



$$\sigma_{ex}^{N} = \sigma_{nom}^{N} = 2,05 \cdot \frac{4 \cdot 892}{\pi \cdot 42^2} = 1,3 \text{ MPa}$$

It is evident that the maximum stress value σ_{ex}^{N} created by the normal force is negligible against the value $\sigma_{ex}^{M_o}$ created by the bending moment. For the evaluation of limit states, the use of stresses created by flexion is sufficient.

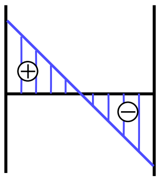
Evaluation of the maximum bending moment M_o:

$$\frac{\mathrm{d}M_o(\varphi)}{\mathrm{d}\varphi} = 0 \rightarrow F_{Bz}R\cos\varphi^* - F_{Bx}R\sin\varphi^* = 0 \rightarrow \varphi^* = 67^o \rightarrow M_{o_{ex}} = M_o(\varphi^*) = 590,7\,\mathrm{Nm}$$

$$M_{o_{\max}} = M_{oA} = |-757|\ \mathrm{Nm}$$

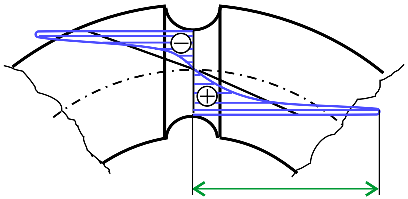
Possible dangerous sections: 1. fixed support (maximum bending moment)
2. notch location $M_o(\varphi = \pi/2) = 892 \cdot 1 - 379 \cdot 1 =$ notch
513 Nm

1. fixed support



$$\sigma_{M_o \max} = \frac{M_o}{W_o} = \frac{32 \cdot 757 \cdot 10^3}{\pi \cdot 50^3} = 61,7 \text{ MPa}$$

2. notch



$$\sigma_{ex}^{Mo} = \alpha \sigma_{nom}^{Mo} = 1,9 \cdot \frac{32 \cdot 513 \cdot 10^3}{\pi \cdot 42^3} = 134 \text{ MPa}$$

The safety factor against the limit state of elasticity $k_K = \dfrac{\sigma_K}{\sigma_{\max}} = \dfrac{195}{134} = 1,5.$