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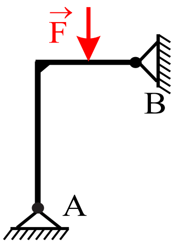
Analysis:

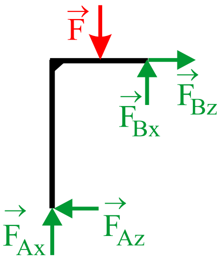


Classification of the bar: in-plane problem, the bar created of two straight parts - angular bar, supported, loaded by an external force.

angular bar

supports

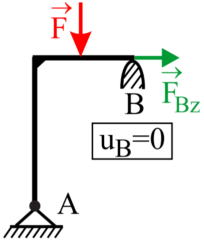
Free body diagram:



Statical analysis:
$\mu = 4, \quad \nu = 3$ (general in-plane system of forces)
$s = \mu - \nu = 1 \quad \Rightarrow$ the problem is onefold statically indeterminate

statical analysis

Released structure:



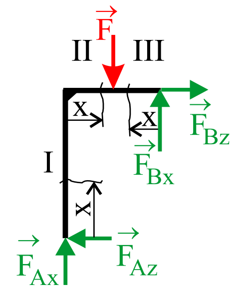
We solve the **equations of static equilibrium** for support resultants, we obtain them as functions of the statical redundant F_{Bz}:

equations of static equilibrium

$$\sum F_x = 0: \quad F_{Ax} + F_{Bx} - F = 0$$
$$\sum F_z = 0: \quad F_{Az} - F_{Bz} = 0$$
$$\sum M_A = 0: \quad F_{Bx} 2a - F_{Bz} b - Fa = 0$$

$$F_{Bx} = \frac{F_{Bz} b}{2a} + \frac{F}{2}, \quad F_{Ax} = F - F_{Bx} = \frac{F}{2} - \frac{F_{Bz} b}{2a}, \quad F_{Az} = F_{Bz}$$

Distribution of inner resultants:



$$
\begin{array}{llcl}
x \in (0; b) & N^{I}(x) & = & -F_{Ax} = \frac{F_{Bz} b}{2a} - \frac{F}{2} \\
 & T^{I}(x) & = & F_{Az} = F_{Bz} \\
 & M_o^{I}(x) & = & F_{Az} x = F_{Bz} x \\
 & & & \\
x \in (0; a) & N^{II}(x) & = & F_{Az} = F_{Bz} \\
 & T^{II}(x) & = & F_{Ax} = \frac{F}{2} - \frac{F_{Bz} b}{2a} \\
 & M_o^{II}(x) & = & F_{Ax} x + F_{Az} b = \\
 & & = & \frac{F}{2} x - \frac{F_{Bz} b}{2a} x + F_{Bz} b \\
x \in (0; a) & N^{III}(x) & = & F_{Bz} \\
 & T^{III}(x) & = & -F_{Bx} = -\frac{F_{Bz} b}{2a} - \frac{F}{2} \\
 & M_o^{III}(x) & = & F_{Bx} x = \frac{F_{Bz} b}{2a} x + \frac{F}{2} x
\end{array}
$$

Modification of the compatibility equation and calculation of the support resultants:

The bar is loaded by combination of tension, shear and flexion. It can be derived that the bending moment is the only significant component of inner resultants, if the length of the bar is at least one order higher than the dimensions of the cross section. (The contribution of $\vec{N}$ and $\vec{T}$ to the total deformation is negligible against the contribution of the bending moment $\vec{M}_o$).

We use Castigliano's theorem to modify the compatibility equation:

Castigliano's theorem

$$
\begin{aligned}
u_B &= \frac{\partial W}{\partial F_{Bz}} = \int\limits_0^{\pi} \frac{M_o(\varphi)}{EJ} \frac{\partial M_o(\varphi)}{\partial F_B} \mathrm{d}s = \\
&= \frac{1}{EJ} \left[\int\limits_0^{b} F_{Bz} x^2 \mathrm{d}x + \int\limits_0^{a} \left(\frac{F}{2} x - \frac{F_{Bz} b}{2a} x + F_{Bz} b \right) \left(b - \frac{bx}{2a} \right) \mathrm{d}x + \right. \\
&\left. + \int\limits_0^{a} \left(\frac{F_{Bz} b}{2a} x + \frac{F}{2} x \right) \frac{bx}{2a} \mathrm{d}x \right] = 0
\end{aligned}
$$

$$
F_{Bz} = -\frac{3a^2 b}{4b^3 + 8ab^2} F = -100 \text{ N}
$$

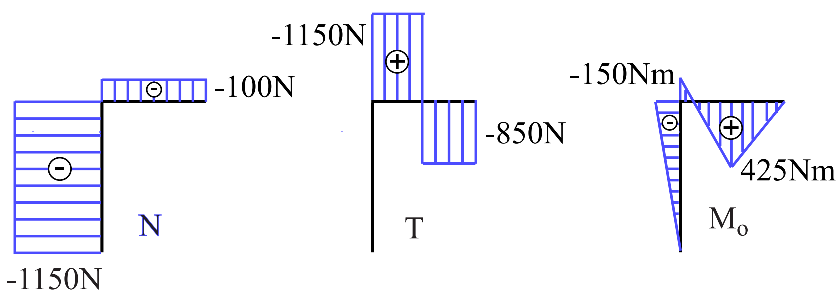
Check for the limit state of elasticity:

limit state

We are about to calculate the safety factor against the limit state of elasticity so that we need to find the dangerous section (i.e. the section with minimum inner resultants or with a stress concentration) and the dangerous point in this section.

dangerous section

We substitute the calculated support resultant in the equations for distribution of inner resultants and calculate their extreme values.



The most significant type of loading is flexion (if you use the formulas for stress in tension $\sigma_N = \frac{N}{S} = \frac{4N}{\pi d^2}$ and in flexion $\sigma_{\max} = \frac{M_o}{W_o} = \frac{32M_o}{\pi d^3}$, you can verify the relation $\sigma_N \ll \sigma_{\max}$). The dangerous section is in the location of the maximum bending moment: $M_{o\max} = 425\,\mathrm{Nm}$. The highest stress value is achieved in the points most distant from the neutral axis; the stress magnitude in these points equals:

$$\sigma_{M_{oex}} = \frac{M_o}{W_o} = \frac{32M_{oex}}{\pi d^3} = 67,6 \cdot 10^6\,\mathrm{Pa}$$

σ_{M_oex}

W_o

The safety factor against the limit state of elasticity is $k_K = \frac{\sigma_K}{\sigma_{\max}} = \frac{300}{67,6} = 4,4.$