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limit state of elasticity
displacement
system with bars
statical analysis

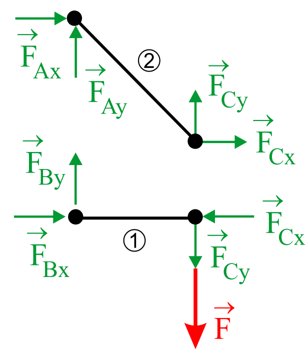
Analysis:

Objectives: check of the limit state of elasticity and of the displacement of the joint C.

Classification: a system of two straight bars loaded in their joint, each of the bars supported by the base, all the supports modelled as pin supports.

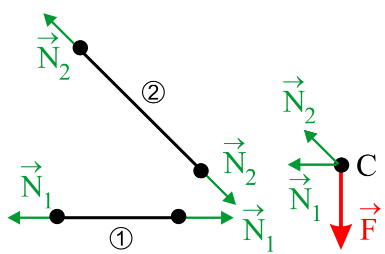
Free body diagram:

a) we isolate all the members as free bodies



Statical analysis:
$\nu = 6, \nu_F = 4, \nu_M = 2$
$\mu = \mu_F = 6$ (two general in-plane systems of forces)
$s = \mu - \nu = 0 \quad \Rightarrow \quad$ statically determinate problem

b) we isolate all members as free bodies and apply our knowledge of statics

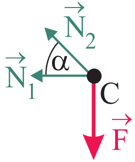


We turn profit from our knowledge on statics to make the free body diagram simpler: All the bars are binary unloaded members ⇒ the static equilibrium can be achieved only if both the support resultants act in oppposite orientations in the same line and if they are equal in magnitude. The line of action of support resultants of any of the bars is thus identical with the centreline of the bar in question.

Pro The free body diagram of the joint C is sufficient for the solution:

Statical analysis: $\nu = 2$, $\mu = 2$ (central in-plane system of forces (intersecting each other in one point))

$s = \mu - \nu = 0 \quad \Rightarrow \quad$ statically determinate problem



Applicable conditions of static equilibrium:

$$F_x: \quad N_1 + N_2 \cos\alpha = 0$$
$$F_y: \quad N_2 \sin\alpha - F = 0$$

statical analysis

Forces in the bars:

$$N_2 = \frac{F}{\sin\alpha}, \qquad N_1 = -N_2 \cos\alpha = -\frac{F}{\mathrm{tg}\ \alpha}.$$

Distribution of stresses:

stress

Since each of the bars is loaded in simple tension (compression) by the normal force constant along the bar centreline, the stress is constant in each of them and can be calculated using the following formula:

$$\sigma(x) = \frac{N(x)}{S(x)}: \qquad \sigma_1 = \frac{N_1}{S_1} = -\frac{F}{S_1\ \mathrm{tg}\ \alpha}, \quad \sigma_2 = \frac{N_2}{S_2} = \frac{F}{S_2 \sin\alpha}.$$

Check for the limit state of elasticity:

limit state

We determine safety factors in all the bars of the system and the lowest of the calculated values is the safety factor against the limit state of elasticity valid for all the system:

$$k_{K1} = \frac{\sigma_{K1}}{\sigma_1}, \quad k_{K2} = \frac{\sigma_{K2}}{\sigma_2}, \quad k_K = \min\{k_{K1}, k_{K2}\}.$$

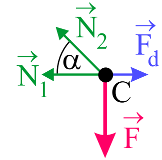
Displacement of the point C:

We will use Castigliagno's theorem, use of other formulas would be much more complex if the bars are not parallel to coordinate axes. Castigliagno's theorem

We are able to calculate the displacement of a point in the direction of an external force acting upon the structure in this point. In our problem, there is a vertical force $\vec{F}$ acting in the point in question but there is no force in the horizontal direction. To calculate also the horizontal displacement of the point C, we need to introduce an additional (horizontal) force $\vec{F}_d$ in this point; its magnitude must equal zero, otherwise the load would not correspond to the original problem.

Note: It is advantageous to carry out this step in the beginning of the solution; now we are forced to create a new free body diagram with the additional force and to repeat the calculation of all the forces.

The new free body diagram of the joint C



static equilibrium

Applicable conditions of static equilibrium:

F_x : $N_1 + N_2 \cos\alpha - F_d = 0$

F_y : $N_2 \sin\alpha - F = 0$

applicable conditions

Forces in the bars:

$$N_2 = \frac{F}{\sin\alpha}, \qquad N_1 = F_d - N_2\cos\alpha = F_d - \frac{F}{\mathrm{tg}\ \alpha}.$$

Strain energy of the i-th bar of the length l_i loaded in simple tension is

strain energy

$$W^{(i)} = \int_0^{l_i} \frac{N_i^2(x)}{2E_iS_i(x)}\mathrm{d}x = \frac{N_i^2 l_i}{2E_iS_i},$$

because $N(x)$ and $S(x)$ are constant along any of the bars in our example.
Strain energy of the whole system consisting of n bars is

$$W = \sum_{i=1}^{n} W^{(i)} = \sum_{i=1}^{n} \frac{N_i^2 l_i}{2E_iS_i}$$

and the displacement u_K of the point of action of the isolated force $\vec{F}_K$ in its direction is

$$u_K = \frac{\partial W}{\partial F_K} = \sum_{i=1}^{n} \frac{N_i l_i}{E_iS_i}\frac{\partial N_i}{\partial F_K}.$$

Horizontal displacement u of the joint C

$$u = \frac{\partial W}{\partial F_d} = \sum_{i=1}^{2} \frac{N_i l_i}{E_i S_i} \frac{\partial N_i}{\partial F_d} = \frac{(F_d - \frac{F}{\operatorname{tg}\alpha}) l_1}{E_1 S_1} 1 + \frac{F l_2}{E_2 S_2 \sin\alpha} \cdot 0 = -\frac{F l_1}{E_1 S_1 \operatorname{tg}\alpha} = -\frac{F h}{E_1 S_1 \operatorname{tg}^2\alpha}$$

Vertical displacement v of the joint C

$$\begin{aligned} v &= \frac{\partial W}{\partial F} = \frac{(F_d - \frac{F}{\operatorname{tg}\alpha}) l_1}{E_1 S_1} \left(-\frac{1}{\operatorname{tg}\alpha}\right) + \frac{F l_2}{E_2 S_2 \sin\alpha} \frac{1}{\sin\alpha} = \frac{F h}{E_1 S_1 \operatorname{tg}^3\alpha} + \frac{F h}{E_2 S_2 \sin^3\alpha} = \\ &= \frac{F h}{\sin^3\alpha} \left(\frac{\cos^3\alpha}{E_1 S_1} + \frac{1}{E_2 S_2} \right) \end{aligned}$$

Total displacement δ_C of the joint C

can be calculated as vector summation of both of the above components

$$\delta_C = \sqrt{u^2 + v^2}$$