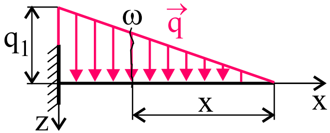
Back to problem analysis

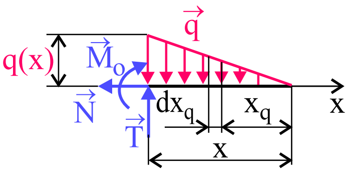
Classification of the bar: straight, loaded by external forces, supported.
Statical analysis: $s = \mu - \nu = 3 - 3 = 0.$



The bar has a free end ⇒ we need not to calculate the reactions, we can isolate elements containing this free end. We need not to divide the bar into intervals, because there is no change in the character of distributed loads along the centreline.
We evaluate the components of inner resultants from the equations of static equilibrium of the isolated finite element.

Integral approach:

integral approach



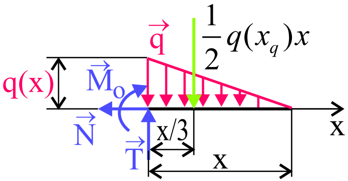
$q(x_q) = \frac{q_1}{l} x_q$

$$F_x : \quad N(x) = 0$$

$$F_z : \quad T(x) = \int_0^x q(x_q)\mathrm{d}x_q = \int_0^x \frac{q_1}{l} x_q \mathrm{d}x_q = \frac{q_1 x^2}{2l}$$

$$M_y : \quad M_o(x) = -\int_0^x q(x_q)(x - x_q)\mathrm{d}x_q = -\int_0^x \frac{q_1}{l} x_q (x - x_q)\mathrm{d}x_q = -\frac{q_1 x^3}{6l}$$

The shear force and the bending moment can be evaluated in an easier way if we turn profit of our knowledge of statics. The magnitude of the (statically equivalent) resultant of the distributed load (created by mutually parallel in-plane forces) equals to the area of the diagram of the distributed load and it acts in the centre of gravity of this diagram.



$$F_z: \quad T(x) = \frac{1}{2}q(x)x = \frac{q_1 x^2}{2l}$$

$$M_y: \quad M_o(x) = -\frac{1}{2}q(x)x\frac{x}{3} = -\frac{q_1 x^3}{6l}$$

differential approach

Differential approach:

Graphical representation of the distribution of components of inner resultants:

$x = 0: \quad N = 0;\ T = 0;\ M_o = 0$

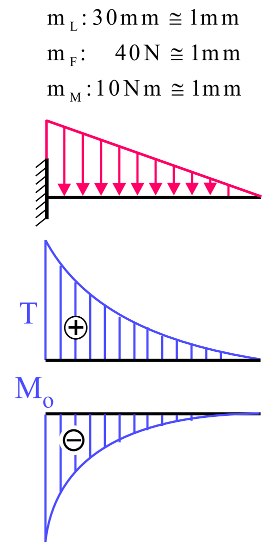
$x = l: \quad N = 0;\ T = 640\,\mathrm{N};\ M_o = -170,7\,\mathrm{Nm}$

Shear force:

$q_T(x) > 0$ (linear) $\rightarrow \quad T(x)$ quadratic curve (parabola)

Bending moment:

function $T(x)$ is parabolic $\rightarrow \quad M_o(x)$ is a cubic parabola.



rules