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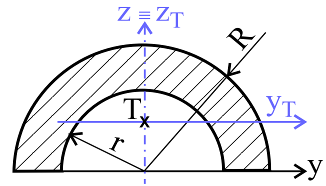
principal c.s.

The principal central axes are characterized, in addition to their point of intersection in the centre of gravity, by a zero deviatoric quadratic moment related to them. One of the basic properties of the second moments is that the deviatoric moment is always zero if at least one of the axes it is related to is an axis of symmetry of the section. Therefore the axis of symmetry is identical with one of the principal central axes and the latter is perpedicular to it in the centre of gravity. In this way, the principal central axes are determined.

The section has a shape of a half annulus; we can turn profit of the relations valid for quadratic moments of a circle related to the axes y and z (see figure).



basic shapes properties

It results from the additivity of quadratic moments (the quadratic moment of the section equals the sum of quadratic moments of its parts if related to the same axes) that the quadratic moment of a half circle is a half of the quadratic moment of the circle (to the same axis). The same property can be used to calculate the quadratic moments of an annulus or, in our example, semiannulus:

$$J_y = J_z = \frac{1}{2}\left(\frac{\pi R^4}{4} - \frac{\pi r^4}{4}\right) = \frac{\pi}{1}\left(R^4 - r^4\right) = 2\ 136 \cdot 10^3\,\mathrm{mm}^4$$

As we are about to calculate the central quadratic moments, we need to calculate the position of the centre of gravity::

centre of gravity

$$y_T = 0, \qquad z_T = \frac{\sum z_i S_i}{\sum S_i} = \frac{\frac{4R}{3\pi}\frac{\pi R^2}{2} - \frac{4r}{3\pi}\frac{\pi r^2}{2}}{\frac{\pi R^2}{2} - \frac{\pi r^2}{2}} = \frac{4}{3\pi}\frac{R^2 + Rr + r^2}{R + r} = 26\,\mathrm{mm}$$

Then we will transform the quadratic moments by translation of the axes into the centre of gravity using Steiner's theorem:

Steiner's theorem

$$J_y = J_{y_T} + (-z_T)^2\, S \quad \Rightarrow \quad J_{y_T} = J_y - (-z_T)^2\,\frac{\pi}{2}\left(R^2 - r^2\right) = 437,31 \cdot 10^3\,\mathrm{mm}^4 = J_2$$

$$J_{y_T} = J_y = 2\ 136 \cdot 10^3\,\mathrm{mm}^4 = J_1$$