Back to problem

Analysis:

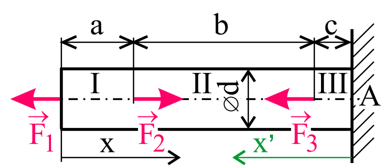
Objectives: presentation of the bar loaded in tension (compression), if the inner resultant $N(x)$ varies along the centreline.

variability of inner resultant

Classification of the bar: straight supported bar, loaded by forces (stepwise changes of inner resultant in isolated points).

Resultants consisting of three force and three moment components can be created in the (fixed in 3D space) support A. It results from the static equilibrium that all the moments and transversal forces in the support A equal zero (if the gravitational forces are negligible). Thus the support can be replaced by a force acting in the bar centreline.

As one of the ends of the bar is not supported, we do not need to calculate the support resultants. We divide the bar into three intervals in the discontinuity points of the normal force, created by the isolated loads:



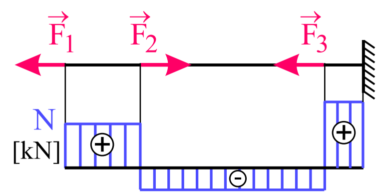
supported beam

Statical analysis:

$\nu = 1, \quad \mu = 1$ (a system of forces acting in the same line)

$s = \mu - \nu = 0 \quad \Rightarrow \quad$ statically determinate system

analysis



inner resultant

Distribution of the inner resultant:

$$x \in (0, a): \qquad N_I = F_1 = 40\,\mathrm{kN}$$

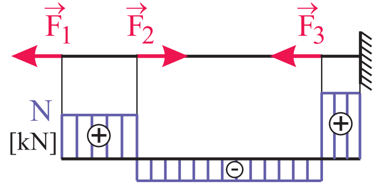
$$x \in (a, a+b): \qquad N_{II} = F_1 - F_2 = -20\,\mathrm{kN}$$

$$x \in (a+b, a+b+c): \qquad N_{III} = F_1 - F_2 + F_3 = 60\,\mathrm{kN}$$

Calculation of stresses:

stress

The bar is loaded in simple tension (compression) in the individual intervals; the only non-zero stress component is the normal stress σ, constant across in the section (of magnitude $S = \frac{\pi d^2}{4}$).



$$x \in (0, a): \qquad \sigma_I = \frac{N_1}{S} = 127,3\,\mathrm{MPa}$$

$$x \in (a, a+b): \qquad \sigma_{II} = \frac{N_2}{S} = -63,7\,\mathrm{MPa}$$

$$x \in (a+b, a+b+c): \qquad \sigma_{III} = \frac{N_3}{S} = 191\,\mathrm{MPa}$$

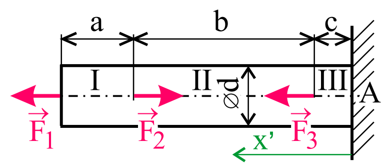
Note: the results valid for the solved computational model will not be valid in reality. The real loads cause always a violation of bar assumptions in the points of action of the loads (e.g. a notch effect).

bar assumptions

Evaluation of displacements:

displacement

$$u(x) = \int_{x_m}^{x} \frac{N(x)}{ES(x)} \mathrm{d}x,$$



where x_m denotes the coordinate of the point of the centreline having zero displacement.

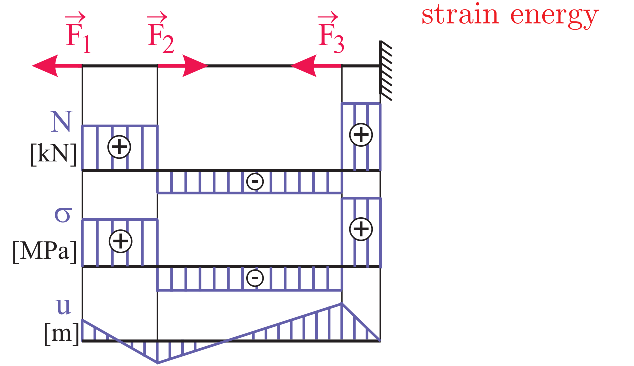
To simplify the evaluation of displacements, we can introduce another coordinate system (x'), having its origin in the fixed support, where the displacement equals zero. As the $N(x)$ is constant per parts, we can easily evaluate the displacements in the individual intervals (the length units are meters):

$$x \in (3;4): \qquad u_I(x') = \frac{N_I \cdot (x'-3)}{ES} + \frac{N_{II} \cdot 2,5}{ES} + \frac{N_{III} \cdot 0,5}{ES} = 6,06 \cdot 10^{-4}(x'-3,5)$$
$$x \in (0,5;3): \qquad u_{II}(x') = \frac{N_{II} \cdot (x'-0,5)}{ES} + \frac{N_{III} \cdot 0,5}{ES} = 3,03 \cdot 10^{-4}(2-x')$$
$$x \in (0;0,5): \qquad u_{III}(x') = \frac{N_{III} \cdot x'}{ES} = 9,09 \cdot 10^{-4}x'$$

The distribution of displacements along the centreline is linear, we calculate the values of displacements in the boundary points of the intervals for a graphical representation of the dependence $u(x')$:

$$u_I(x'=4) = 3,03 \cdot 10^{-4}\,\mathrm{m}, \quad u_{II}(x'=3) = -3,03 \cdot 10^{-4}\,\mathrm{m}, \quad u_{III}(x'=3) = 4,55 \cdot 10^{-4}\,\mathrm{m},$$
$$u_I(x'=3) = 3,03 \cdot 10^{-4}\,\mathrm{m}, \quad u_{II}(x'=0,5) = 4,55 \cdot 10^{-4}\,\mathrm{m}, \quad u_{III}(x'=0) = 0$$

Evaluation of the strain energy of the bar:



$$W_{\Omega_0} = \int_0^l \frac{N^2(x)}{2ES}\mathrm{d}x$$

As the $N(x)$ is constant per parts, the total strain energy can be calculated as a summation of strain energies in the individual intervals of the bar (SI units used):

$$W = \frac{N_I^2 \cdot 1}{2ES} + \frac{N_{II}^2 \cdot 2,5}{2ES} + \frac{N_{III}^2 \cdot 0,5}{2ES} = 33,35\,\mathrm{J}$$