Back to problem

Analysis:

angular bar

Classification of the bar: in-plane problem, angular bar, supported, loaded by temperature load.

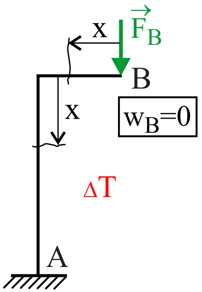
Supports:	A	fixed support	$\mu_A = 3,$
	B	roller support (jen 1 funkční)	$\mu_B = 1$

Statical analysis:

statical analysis

$\mu = 4, \quad \nu = 3$ (general in-plane system of forces)

$s = \mu - \nu = 1 \quad \Rightarrow \quad$ the problem is onefold statically indeterminate



Released structure:

Distribution of the bending moment M_o:

$$\begin{aligned} N^I(x) &= 0, & T^I(x) &= F_B, & M_o^I(x) &= -F_B \cdot x, \\ N^{II}(x) &= -F_B, & T^{II}(x) &= 0, & M_o^{II}(x) &= -F_B \cdot l_2 \end{aligned}$$

Modification of the compatibility equation and calculation of support reactions:

The bar is loaded by combination of tension, shear and flexion. It can be derived that the bending moment is the only significant component of inner resultants, if the length of the bar is at least one order higher than the dimensions of the cross section.

Example 627

$$\begin{aligned} w_B^{F_B} &= \frac{\partial W}{\partial F_B} = \int_\gamma \frac{M_o}{EJ} \cdot \frac{\partial M_o}{\partial F_B} \mathrm{d}s = \\ &= \frac{1}{EJ} \left\{ \int_0^{l_2} -F_B x(-x) \mathrm{d}x + \int_0^{l_1} -F_B l_2(-l_2) \mathrm{d}x \right\} = \frac{F_B}{EJ} \left\{ \frac{l_2^3}{3} + l_1 l_2^2 \right\} \\ w_B &= w_B^{F_B} + w_B^{\Delta T} = \frac{F_B}{EJ} \left\{ \frac{l_2^3}{3} + l_1 l_2^2 \right\} - \alpha l_1 \Delta T = 0 \end{aligned}$$

Note: the negative sign of the displacement created by temperature change results from the opposite orientation of the force $\vec{F}_B$ and of the elongation created by the temperature change. The (temperature) elongation of the horizontal part l_2 of the bar is not significant in this problem, because the horizontal displacement is not restricted by the supports.

Castigliano's theorem

We solve the compatibility equation for the **resultant in the support B:**

$$F_B \left(\frac{l_2^3}{3} + l_1 l_2^2 \right) = EJ\alpha l_1 \Delta T \quad \Rightarrow \quad F_B = \frac{EJ\alpha l_1 \Delta T}{\frac{l_2^3}{3} + l_1 l_2^2}$$

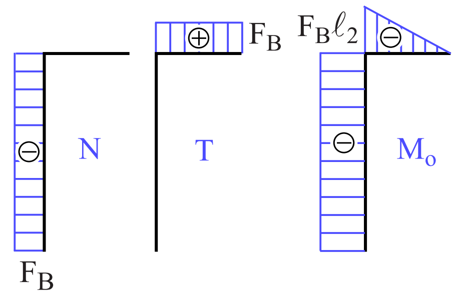
Check for the limit state of elasticity:

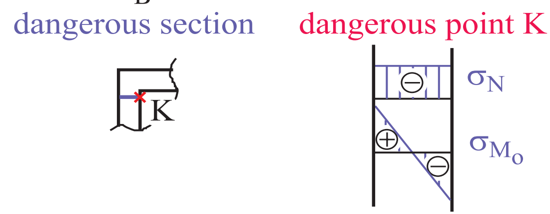
limit state

dangerous section

We are about to calculate the safety factor against the limit state of elasticity so that we need to find the dangerous section (i.e. the section with minimum inner resultants or with a stress concentration) and the dangerous point in this section.

We substitute the calculated support resultant into the equations for distribution of inner resultants and calculate their extreme values:





$$\sigma_{max} = \alpha_N \frac{4F_B}{\pi d^2} + \alpha_{M_o} \frac{32 F_B l_2}{\pi d^3}$$

$$k_K = \frac{\sigma_K}{\sigma_{\max}}$$