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Analysis:

limit state of elasticity

Objectives: check of the limit state of elasticity.

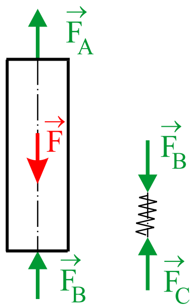
Classification of the bar: straight bar, loaded by forces, supported by one rigid and one flexible (elastic) support.

supported bar

Free body diagram:

support

elastic support



Resultants consisting of three force and three moment components can be created in the (fixed in 3D space) support A. If we ensure a sufficient coaxiality and alignment of the supports, the components without any significant external loads in their direction will be negligible, and the only one significant component of the support resultant will be the force parallel to the bar centreline and thus also to the external loads. The same holds for the flexible (elastic) support B; it shows significant displacements so that it is modelled by a spring characterized by the spring constant c_P. This constant quantifies the linear relation between the displacement and the force in the support, determined by the relation $\Delta l_P = c_P F_P$, where Δl_P is the deformation displacement of the support and F_P is the force acting upon the support.

statical analysis

Statical analysis: $\nu = 1, \qquad \mu = 2$ (a system of forces acting in the same line)

$s = \mu - \nu = 1 \quad \Rightarrow$ statically onefold indeterminate system

Applicable conditions of static equilibrium:

$$F_x : \quad F_A - F + F_B = 0$$

Released structure:

released structure

a) The force-dependent compatibility equation $u_B^{(2)} = F_B c$

force dependent condition

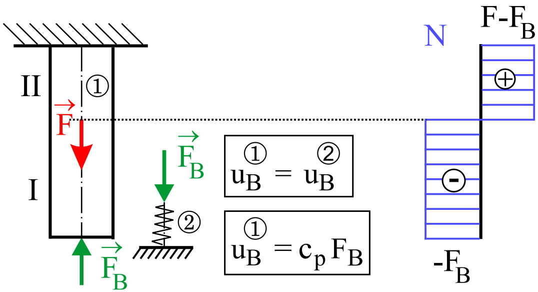
Distribution of the inner resultant:

$x \in (0, b): \quad N_I = -F_B$

$x \in (0, a): \quad N_{II} = F - F_B$

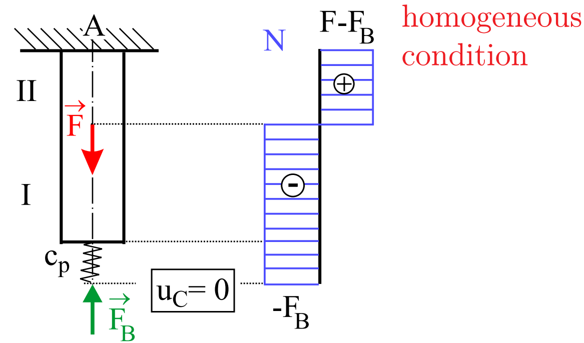
Modification of the compatibility equations:

$$u_B^{(1)} = -\frac{F_B}{ES} b + \frac{F - F_B}{ES} a = u_B^{(2)} = F_B c_p$$



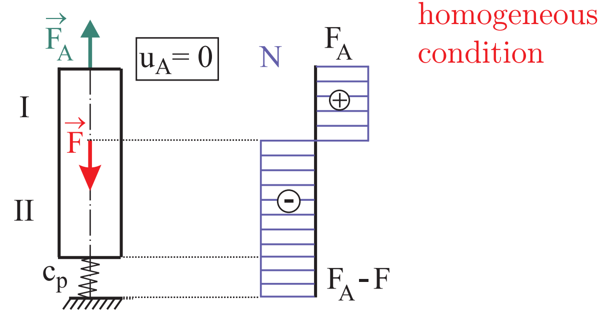
The positive sign in the right-hand side of the equation results from the identical orientation of the positive displacements of the bar and the spring; positive displacement of the point B of the bar is dawnwards (elongation) as well as the positive displacement of the end of the spring (positive in the direction of the acting force F_B). The sign, however, would be negative if Castigliano's theorem is used!

b) The compatibility equation $\quad u_C = 0$



$$u_C = -F_B c_p - \frac{F_B}{ES} b + \frac{F - F_B}{ES} a = 0$$

c) The compatibility equation $\quad u_A = 0$



Distribution of the inner resultant:

$x \in (0, b): \quad N_I = F_A$

$x \in (0, a): \quad N_{II} = F_A - F$

Modification of the compatibility equations:

$$u_A = \frac{F_A}{ES} a + \frac{F_A - F}{ES} b + (F_A - F) c_p = 0$$

$$F_A = F - F_B \quad \Rightarrow \quad u_A = \frac{F - F_B}{ES} a - \frac{F_B}{ES} b - F_B c_p = 0$$

All the three released structures result in the same form of the compatibility equation.

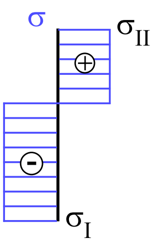
Set of equations:

$$F_A - F + F_B = 0 \qquad \frac{F - F_B}{ES} a - \frac{F_B}{ES} b - F_B c_p = 0$$

Solution:

$$F_B = \frac{Fa}{a + b + c_p ES}, \qquad F_A = F - F_B = \frac{Fb + F c_p ES}{a + b + c_p ES}.$$

Distribution of stresses: stress



Since the bar is loaded in simple tension (compression), the stress is constant across the section and can be calculated using the following formula:

$$\sigma(x) = \frac{N(x)}{S(x)}: \qquad \sigma_I = \frac{N_I}{S} = \frac{-F_B}{S}, \quad \sigma_{II} = \frac{N_{II}}{S} = \frac{F_A}{S}$$

Check for the limit state of elasticity: limit state

We determine the section with extreme stress (dangerous section) from the stress distribution and calculate the safety factor against the limit state of elasticity:

$$k_K = \frac{\sigma_K}{\sigma_{\max}}.$$

Since the temperature changes and production inaccuracies are negligible, the check of the assemblage state (without external loads) is not needed.