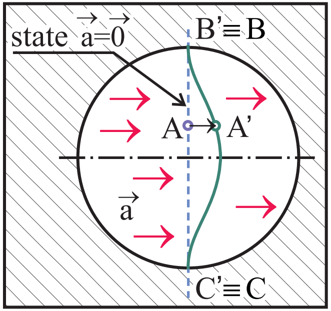
5. Deformation of bodies

Deformation as a component of mechanical motion was mentioned already in statics. Under deformation we usually understand a change in the shape and dimensions of the body. Deformation can be described by changes in distances between any two points of the body and by changes in angles defined by any three points of the body, under condition that the continuity of the body is not violated.

deformation

Not all these changes, however, can be observed because we can see only the surface of the body. In some practical cases deformation can occur only inside the body without any significant changes in the shape and dimensions of the body.



Therefore the deformation of the body must be defined in a more general way:

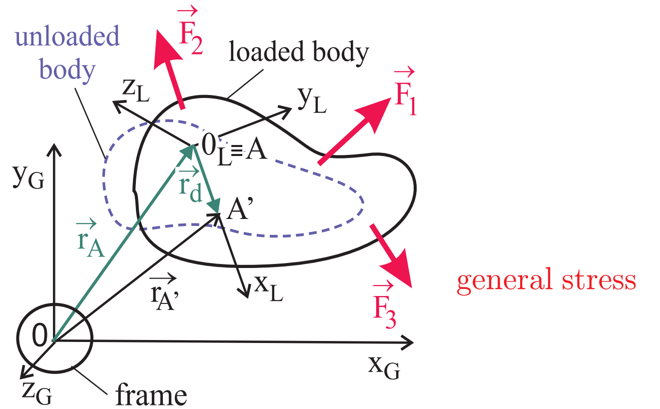
Deformation of the body consists of changes of shape and dimensions of the body and of any element of the body defined in the undeformed state.

For the mathematical description of the deformation, we need to describe the location of any point of the body by its position vector in the unloaded (initial) as well as in the loaded (deformed) states.

Usually two different (mostly cartesian) coordinate systems are used for the description:

a) **global coordinate system** – its origin is joined with the frame,

b) **local coordinate system** – its origin is joined with any chosen point of the investigated body and the axes are oriented with respect to the problem to be solved. (For example, such a coordinate system, with one axis oriented normal to the investigated section, was used for decomposition of general stress $\vec{f}$ into its normal and shear components.)



A change of position vector of any point of the body means displacement of the point; if the body as a whole does not move with respect to the global coordinate system (in which position vectors are defined), this displacement is caused by the deformation of the body, it is the so called **deformation displacement**. When the body is fixed to the frame, any displacement ($\vec{r}_d = \vec{r}_{A'} - \vec{r}_A$) of a point of the body represents the deformation of the body.

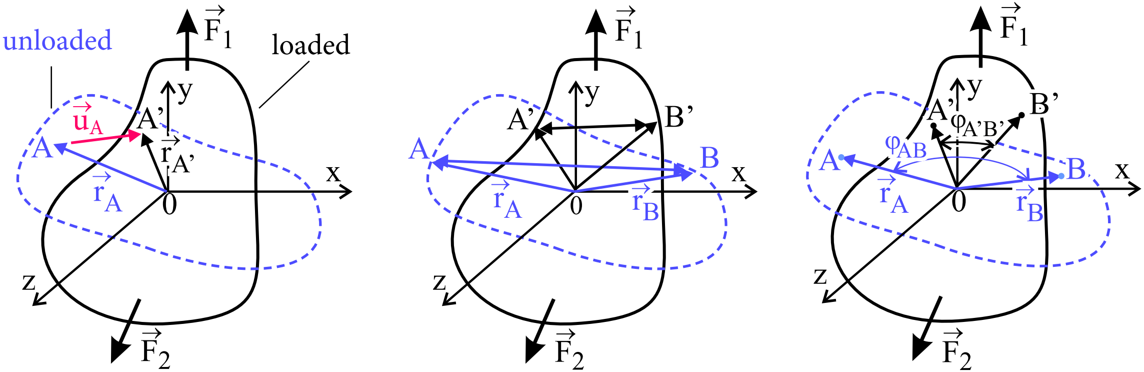
Displacement of a point is determined by the **change of its position vector.**

If the displacements of two points differ, their distance changes in the consequence of deformation; this change of length can be calculated by subtraction of their displacement vectors. If any angle is defined within the body using its three points, then the change of this angle can be calculated (by means of vector algebra) using displacements vectors of these points. It can be generalised that any deformation parameter can be calculated from displacement vectors of all the points of the body.

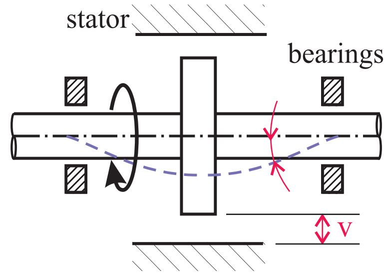
Deformation of the body is unambiguously determined by the set of displacement vectors of all its points.

Displacements of points of the body represent its **basic deformation characteristics**, enabling us to determine length and angle changes within the body.

displacement	**length changes**	**angle changes**
position change of a point of the body	change of the distance between two points	of the angle given by three points of the body



Deformation can be observed and measured in a limited range only. The limitation lies in the fact that in practice we are able to measure only a limited number of **deformation characteristics of the body**.

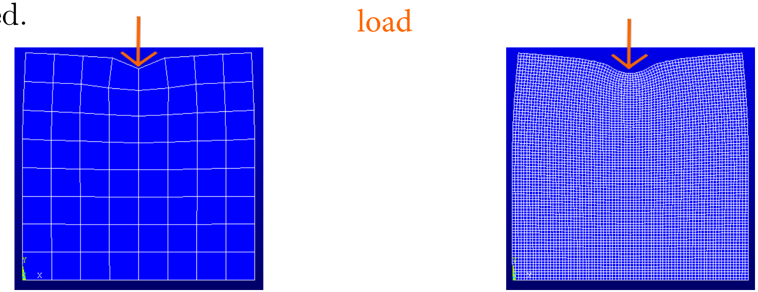


To judge the deformation of the body, we do not need to know all the deformation parameters; it is sufficient to evaluate only the parameters important from the viewpoint of the body function.
For example, at a rotating shaft with a wheel, it is important to evaluate:

- deflection of the shaft in the location of the rotor,
- deflection angle in the location of bearing,
- change of the rotor diameter caused by centrifugal forces.

Deformation of the body differs generally in each of its points; to describe the local deformation a quantity called **deformation in a point of the body** is introduced..

This quantity can be deduced using the virtual experiment in the figure. A regular rectangular network is drawn on the surface of a highly deformable body; it deforms together with the body when loaded.

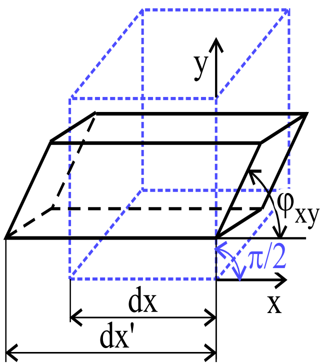


If the network is rough and the deformation of the body non-homogeneous, each of the squares drawn on the body surface has another shape after deformation. If we smooth up the network (diminish the network element size), we can come to a stage in which the neighbouring squares will be nearly identical even in the deformed state. Deformation inside the individual square will then be nearly homogeneous, i.e. the same in all of its points. The non-homogeneity of the deformation field decides how smooth the network must be to achieve this state. This state can be achieved quite exactly only by infinitesimal size of the square. As in the 3D space every square corresponds to a cubic element, deformation of this elementary cube can be identified with the deformation in any point inside this cube.

Deformation of the elementary cube is determined by length changes of its three edges and by changes of three angles between its faces, all of them described by the following formulas:

- length strains (relative changes of lengths):
  $\varepsilon_x = \dfrac{\mathrm{d}x' - \mathrm{d}x}{\mathrm{d}x}, \quad \varepsilon_y = \dfrac{\mathrm{d}y' - \mathrm{d}y}{\mathrm{d}y}, \quad \varepsilon_z = \dfrac{\mathrm{d}z' - \mathrm{d}z}{\mathrm{d}z}$
  ($\varepsilon > 0 \rightarrow$ elongation, $\varepsilon < 0 \rightarrow$ shortening),

- shear (angular) strains (relative changes of right angles):
  $\gamma_{xy} = \dfrac{\pi}{2} - \varphi_{xy}, \quad \gamma_{xz} = \dfrac{\pi}{2} - \varphi_{xz}, \quad \gamma_{yz} = \dfrac{\pi}{2} - \varphi_{yz}.$



These six strains can be (similar to stresses) ordered into a square matrix describing the strain tensor T_ε in the given coordinate system; sometimes it is also called deformation tensor.

$$T_\varepsilon = \begin{pmatrix} \varepsilon_x & \frac{\gamma_{xy}}{2} & \frac{\gamma_{xz}}{2} \\ \frac{\gamma_{yx}}{2} & \varepsilon_y & \frac{\gamma_{yz}}{2} \\ \frac{\gamma_{zx}}{2} & \frac{\gamma_{zy}}{2} & \varepsilon_z \end{pmatrix}$$

Thus **strain tensor** is determined by six strain components, i.e. by three length and three shear strains. Then the following definition can be formulated:

Deformation in a point of the body is a relative deformation (strain) of an infinitesimal element containing this point; it is described by the **strain tensor** T_ε.

Thus the term „**deformation**“ can have two quite different meanings:

Example

1. *deformation displacement* [mm],
2. *strain* – a relative dimensionless quantity.

Checking questions

It is strictly necessary to distinguish exactly between these two meanings.