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system of bars

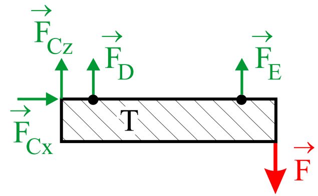
Analysis:

A system of two straight bars, each of them supported by the base and by the body T, all the supports modelled as pin supports.

Free body diagram:

Example 434

The free body diagram of the body T is sufficient for solution:



statical analysis

Statical analysis:
$\mu = 4 \qquad \nu = 3$ (general system of in-plane forces)
$s = \mu - \nu = 1 \quad \Rightarrow \quad$ statically onefold indeterminate problem.

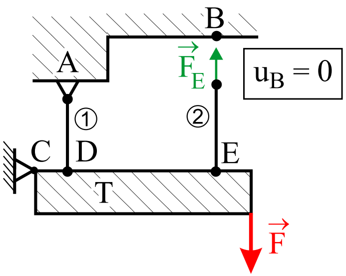
Released structure:

The easiest way of how to create the primary (released) structure is usually to release a joint between one of the bars and the base, because the base does not move and its deformation is negligible ($\rightarrow$ zero displacement).

Castigliano's theorem

For instance, we release the bar No. 2 from the base and introduce the following compatibility equation in point B: $u_B = 0$:

$$u_B = \frac{\partial W}{\partial F_E} = \sum_{i=1}^{2} \frac{N_i l_i}{ES} \frac{\partial N_i}{\partial F_E} = 0.$$



Applicable conditions of static equilibrium:

$$\begin{aligned}
&\sum F_x = 0: && F_{Cx} = 0\\
&\sum F_z = 0: && F_{Cz} + F_D + F_E - F = 0\\
&\sum M_{Cy} = 0: && F(a+b+c) - F_E(a+b) - F_D a = 0
\end{aligned}$$

Support reactions

We express them as functions of the force F_E (statical redundant) which acts in the point B:

$$F_{Cx} = 0, \quad F_{Cz} = F_E \frac{b}{a} - F\frac{b+c}{a}, \quad F_D = F\frac{a+b+c}{a} - F_E\frac{a+b}{a}.$$

Modification of the compatibility equation and calculation of the forces in the bars:

$$u_B = \sum_{i=1}^{2} \frac{N_i l_i}{ES} \frac{\partial N_i}{\partial F_E} = \frac{F\frac{a+b+c}{a} - F_E\frac{a+b}{a}}{E_1 S_1} l_1 \left(-\frac{a+b}{a}\cdot 1\right) + \frac{F_E l_2}{E_2 S_2}\cdot 1 = 0$$

$$N_2 = F_E = \frac{F(a+b+c)}{\frac{a^2}{a+b}\frac{E_1 S_1}{E_2 S_2}\frac{l_2}{l_1} + a + b}, \qquad N_1 = F_D = F\frac{a+b+c}{a} - F_E\frac{a+b}{a}.$$

Distribution of stresses: stress

Since the bars are loaded in simple tension (compression) by constant normal forces, the stresses in each of them are constant and can be calculated using the following formula: $\sigma(x) = \dfrac{N(x)}{S(x)}$:

$$\sigma_1 = \frac{N_1}{S_1} = \frac{Fb}{S_1(a+b)}, \quad \sigma_2 = \frac{N_2}{S_2} = \frac{Fa}{S_2(a+b)}.$$

Check for the limit state of elasticity: limit state

We calculate the safety factor against the limit state of elasticity for both of the bars of the system and the lowest of them will determine the safety factor of the whole system:

$$k_{K1} = \frac{\sigma_{K1}}{|\sigma_1|}, \quad k_{K2} = \frac{\sigma_{K2}}{|\sigma_2|}, \quad k_K = \min\{k_{K1}, k_{K2}\}.$$

Note:
Because of the statical indeterminateness of the system, the solution is valid only under assumption that the production inaccuracies and temperature changes are negligible.