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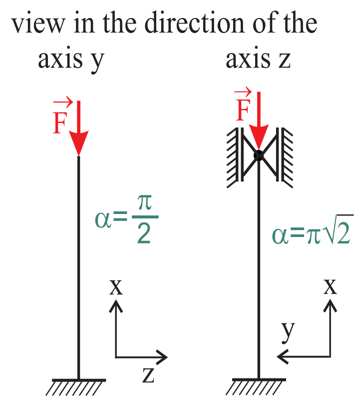
Analysis: The bar is straight, supported and loaded in compression, the problem is three-dimensional. As the external force $\vec{F}$ acts in the bar centreline, the reactions in the top support are zero. The limit state of buckling can occur, if the length of the bar is substantially higher than the dimensions of the cross section.

F_{kr}

J_y, J_z

a) Buckling in the plane (xy)



$$k_{vy} = \frac{F_{kr}}{F} = \frac{\alpha^2 \frac{EJ_z}{l^2}}{F} = \frac{\frac{\pi^2 EJ_z}{4l^2}}{F}$$

Buckling in the plane (xz)

$$k_{vz} = \frac{F_{kr}}{F} = \frac{\alpha^2 \frac{EJ_y}{l^2}}{F} = \frac{\frac{2\pi^2 EJ_y}{l^2}}{F}$$

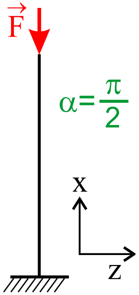
It is required that the safety factor against buckling in the directions of both of the principal central axes be the same; therefore both the critical forces must be equal in magnitude:

$$k_{vy} = k_{vz} \quad \Longrightarrow \quad \frac{\pi^2 EJ_z}{4l^2} = \frac{2\pi^2 EJ_y}{l^2}$$

$$\frac{\pi^2 \frac{ab^3}{12}}{4} = 2\pi^2 \frac{a^3 b}{12} \quad \Longrightarrow \quad \frac{b^2}{4} = 2a^2 \quad \Longrightarrow \quad \frac{a}{b} = \frac{1}{2\sqrt{2}}$$

b) Under compression, limit states of elasticity or buckling can occur. As the geometrical parameters of the cross section are not defined, two cases can come into existence:

slenderness



Buckling in the plane (xy) $\Rightarrow$ $J_2 = J_z = \frac{ab^3}{12}$

Critical slenderness $\lambda_K = \alpha\sqrt{\frac{E}{\sigma_K}} = \frac{\pi}{2}\sqrt{\frac{E}{\sigma_K}}$

Slenderness of the bar $\lambda = \frac{l}{\sqrt{\frac{J_z}{S}}} = \frac{l}{\sqrt{\frac{\frac{ab^3}{12}}{ab}}} = \frac{l}{\sqrt{\frac{b^2}{12}}} = \frac{2l\sqrt{3}}{b}$

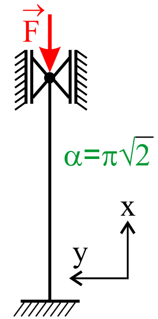
$$\lambda < \lambda_K \longrightarrow k_K = \frac{\sigma_K}{\sigma_{\max}} = \frac{\sigma_K}{\frac{F}{S}} = \frac{ab}{F}\sigma_K$$

$$\lambda > \lambda_K \longrightarrow k_{vy} = \frac{F_{kr}}{F} = \frac{\alpha^2\frac{EJ_z}{l^2}}{F} = \frac{\frac{\pi^2 E\frac{ab^3}{12}}{4l^2}}{F} = \frac{\pi^2 Eab^3}{48l^2 F}$$

Buckling in the plane (xz) $\quad\Rightarrow\quad J_2 = J_y = \dfrac{a^3 b}{12}$

Critical slenderness $\quad \lambda_K = \alpha\sqrt{\dfrac{E}{\sigma_K}} = \pi\sqrt{2}\sqrt{\dfrac{E}{\sigma_K}}$

Slenderness of the bar $\quad \lambda = \dfrac{l}{\sqrt{\dfrac{J_y}{S}}} = \dfrac{l}{\sqrt{\dfrac{\frac{a^3 b}{12}}{ab}}} = \dfrac{l}{\frac{a}{\sqrt{12}}} = \dfrac{2l\sqrt{3}}{a}$



$$\lambda < \lambda_K \quad\longrightarrow\quad k_K = \frac{\sigma_K}{\sigma_{\max}} = \frac{\sigma_K}{\frac{F}{S}} = \frac{ab}{F}\sigma_K$$

$$\lambda > \lambda_K \quad\longrightarrow\quad k_{vz} = \frac{F_{kr}}{F} = \frac{\alpha^2 \frac{EJ_y}{l^2}}{F} = \frac{2\pi^2 \frac{E\frac{a^3 b}{12}}{l^2}}{F} = \frac{\pi^2 E a^3 b}{6 l^2 F}$$

The safety factor will equal the minimum of all the safety factor values calculated for the defined geometry of the bar.