3. Element of the body and stress in a section

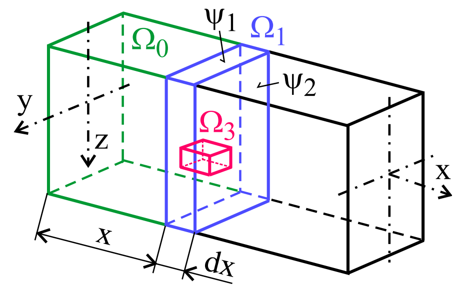
About 200 years ago, Bernoulli came with an idea that in a loaded (and, as a consequence of load, deformed) body **inner forces** are produced; these inner forces exert effort to bring the body into its initial (undeformed) state. One of the basic statements in statics was the following one: if a body is in **static equilibrium** (e.g. motionless in relation to the frame), **any part** of this body must be in static equilibrium as well. The body represented a basic investigated object in statics; in stress analysis, in opposite, we divide the body into parts - elements and we investigate the equilibrium of these elements to evaluate inner forces in the body.

static equilibrium

Element of a body is any continuous part of the body cut off by one ore more virtual sections from the body. Inner forces in the section are forces representing the action of the removed part of the body upon the part that remains, i.e. upon the investigated element.

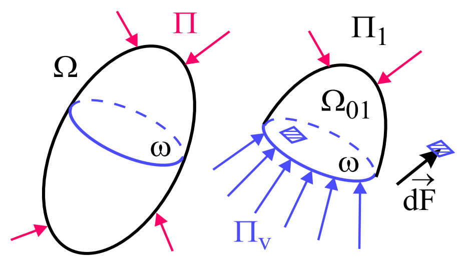
The geometrical shape of the element is chosen with respect to the shape of the body, to the chosen coordinate system and to the character of the problem to be solved. The dimensions of the element can be either finite or infinitesimal.

The element is called finite element



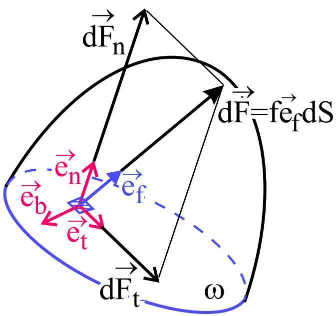
- The element is called **finite element** (Ω_0) if all the dimensions are finite.
- If one of the dimensions is elementary (infinitesimal) the element is called **onefold infinitesimal element** (Ω_1).
- If two or three dimensions of the element are elementary the element is called **twofold** (Ω_2) or **threefold** infinitesimal **element** (Ω_3), respectively.

Investigation of inner forces starts with isolating the element as a free-body. If we cut an element of the body by a single section ω, then the isolation of the element as a free body means, in addition to external forces acting on the element, introducing of *inner forces acting in the section*. These forces are (at least per parts) continuously distributed in the section, i.e. they are specific forces per **unit area**. The above process is called **isolation of an element as a free body**, analogically to the creation of a free body diagram of the whole body in statics. (Free body diagram was used in statics to evaluate external forces acting on the body - i.e. reaction forces in the **supports** of the body.)



force

The elementary force $\mathrm{d}\vec{F} = \vec{f}\,\mathrm{d}S$ (where $\vec{f}$ is a specific force per unit area) acting on the elementary area $\mathrm{d}S$ is called stress or, more **precisely**, **general stress in the section**. The epithet general means that the direction of the stress is general in relation to the investigated section, i.e. it can be perpendicular, parallel or inclined to the section. (The angle as well as the magnitude of the stress can be different at each point of the section.)



It is advantageous to introduce a coordinate system in such a way that one of the axes is perpendicular to the area $\mathrm{d}S$ (parallel to the normal line of the area) and the other(s) will be tangential to this area because the normal stresses di er very substantially from the tangential stresses by their influence on the material behaviour and, **consequently**, on limit states. The general stress $\vec{f}$ can be decomposed in two (or three) components according to the equation:

$$\vec{f} = \sigma\vec{e_n} + \tau\vec{e_t}.$$

The general stress is a vector defined by three components in a 3D space. One of them has normal direction and the two remaining components are tangential to the area $\mathrm{d}S$. The coordinate system can be introduced in an advantageous way so that one of the both tangential components is zero. Then the general stress can be decomposed in two components only, one being perpendicular to the area $\mathrm{d}S$ and called normal stress σ and the other being tangential to this area and called shear (or tangential) stress τ. As only magnitude of the shear stress is significant for the evaluation of limit states, this decomposition in two components is quite sufficient. Then the following equations can be written:

$$\sigma = \vec{f} \cdot \vec{e_n}, \quad \tau = \sqrt{f^2 - \sigma^2} = \vec{f} \cdot \vec{e_t}.$$

Pascal [N/m^2] represents the basic SI unit of general, normal or shear stresses as well as of the specific force per unit area.

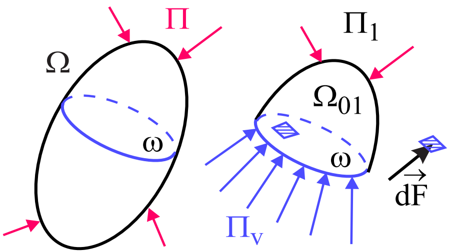
$$[f] = [\sigma] = [\tau] = [\mathrm{d}F/\mathrm{d}S] = \mathrm{Pa}$$

The stress orientation is determined as follows:

normal stress:	$\sigma > 0$	$\Longrightarrow$	tensional, oriented outwards the section,
	$\sigma < 0$	$\Longrightarrow$	compressional, oriented inwards the section,
shear stress:	the orientation can be chosen by convention, the choice is not substantial at isotropic materials.		

orientation

3.1. Principle of stress evaluation



The body Ω is loaded by an equilibrium system of forces Π. If we cut an element Ω_{01} of the body by a virtual section ω, the external forces acting on the element (system of forces Π_1) are not more in static equilibrium. To achieve equilibrium, the system of external forces Π_1 must be **completed (equilibrated)** by internal forces Π_V (general stresses in individual points of the section) acting in the section.

In general we do not know the stress distribution in the section ω, so that the stresses represent an infinite number of variables and the **problem** of stress evaluation is statically indeterminate. The number of applicable static equilibrium conditions cannot be higher than 6, so that a large number of equations are missing for stress evaluation. If we isolate a threefold **infinitesimal** element as a free body, we get a system of partial differential equations with complex boundary conditions. Without use of modern computers this system was not generally solvable, but technical practice required evaluation of stresses and strains in the designed bodies.

number of conditions

static conditions

boundary condition

Therefore approaches came into existence which simplified the problem by some assumptions **formulated on the basis of experimental experience and of the level of science in the contemporary epoch.** These assumptions simplified the solution, but the validity of results was limited, they were valid only for bodies fulfilling these assumptions with a sufficient accuracy. In this way the most simple part of theory of elasticity was established, which uses model bodies for solution and gives results with a limited applicability [2]. An overview of model bodies and at the same time an overview of possibilities of analytical stress analysis is presented in chapter 3.2.

3.2. Overview of analytically solvable model bodies

The problem of stress evaluation can be solved only under certain assumptions. These assumptions define the following types of model bodies:

1. bars (beams, columns),
2. thick wall body cylindrical or spherical,
3. axisymmetric plate,
4. axisymmetric wall (rotating disc),
5. axisymmetric membrane (momentum-free) shell,
6. cylindrical momentum shell.

It is evident from the overview that in addition to the **rod-like** bodies (modelled as bars, beams or columns) only some **axisymmetric bodies** can be solved analytically. For analytical solution, axisymmetry must comprehend not only geometry, but also material, load and couplings must be axisymmetric. Only in these cases also stress and strain states are axisymmetric and they can be solved analytically. Other types of bodies require use of numerical methods and special computer programs. The above types of model bodies are transferred on real bodies which are then called **beams**, **shells**, **plates** etc. Therefore it is noticeable that the model used for stress-strain analysis is not given unambiguously by the shape of the body but it depends on the boundary conditions (e.g. loads and couplings of the body - the same body can be solved as plate or wall in dependence on the load).