Back to problem

Analysis:

A system of six straight slender bars, each of them being joined with at least another bar in both of his ends. All the system is supported by the base in joints only, and loads are acting in joints as well; all the supports can be modelled as pin supports $\Rightarrow$ strut frame.

strut frame

Free body diagram:

statical analysis

free body diagram

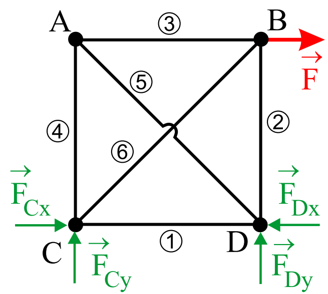
Statical analysis:

$\mu_{ex} = 4, \quad \nu = 3$

$s_{ex} = \mu_{ex} - \nu = 4 - 3 = 1$

$s_{in} = p - (2k - 3) = 6 - (2 \cdot 4 - 3) = 1$

The system is onefold externally and onefold internally statically indeterminate.

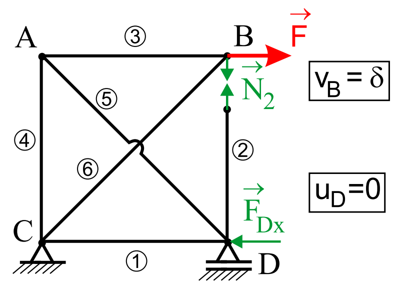


Released structure:

The created released structure should be

a) externally statically determinate:

We change the pin support D into a hinged support and introduce the support reaction $\vec{F}_{Dx}$ (statical redundant) in that direction, in which the displacement is not restricted by this support (in contrast to the original pin support). Further, we formulate the compatibility equation for horizontal displacement in the point D: $u_D = 0$.



b) and internaly statically determinate as well:

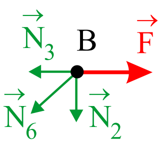
We release the bar No. 2 in the joint B, because this bar shows a production inaccuracy (the solution becomes easier by this choice) and we introduce the compatibility equation for the relative displacement in the point B: $v_B = \delta$.

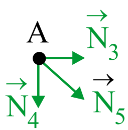
The forces in the bars

can be solved using the successive joint method. We cannot calculate the magnitudes of the forces directly (the system is statically indeterminate) but we express all the forces as functions of the statical redundants ($\vec{F}_{Dx}, \vec{N}_2$). We must start in the joint B, in which only two unknown forces (except for the statical redundants) are acting; we express these unknown forces from the equations of static equilibrium of the joint:

joint method

static equilibrium





$$F - N_6\frac{\sqrt{2}}{2} - N_3 = 0$$

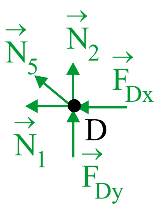
$$N_2 + N_6\frac{\sqrt{2}}{2} = 0$$

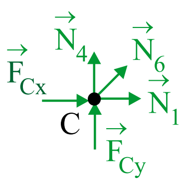
$$N_6 = -\sqrt{2}N_2$$
$$N_3 = F + N_2$$

$$N_3 + N_5\frac{\sqrt{2}}{2} = 0$$

$$N_4 + N_5\frac{\sqrt{2}}{2} = 0$$

$$N_5 = -\sqrt{2}N_2 - \sqrt{2}F$$
$$N_4 = F + N_2$$





$$F_{Dx} + N_1 + N_5\frac{\sqrt{2}}{2} = 0$$

$$F_{Dy} + N_2 + N_5\frac{\sqrt{2}}{2} = 0$$

$$F_{Dy} = F$$
$$N_1 = N_2 + F - F_{Dx}$$

$$F_{Cx} + N_1 + N_6\frac{\sqrt{2}}{2} = 0$$

$$F_{Cy} + N_4 + N_6\frac{\sqrt{2}}{2} = 0$$

$$F_{Cx} = F_{Dx} - F$$
$$F_{Cy} = -F$$

Modification of the compatibility equations and calculation of the forces in the bars:

Castigliano's theorem

$$v_B = \frac{\partial W}{\partial N_2} = \sum_{i=1}^{6} \frac{N_i l_i}{E_i S_i}\frac{\partial N_i}{\partial N_2} = \frac{1}{ES}\Big[(N_2+F-F_{Dx})l\cdot 1 + N_2 l\cdot 1 + (F+N_2)l\cdot 1 +$$
$$+(F+N_2)l\cdot 1 + (-\sqrt{2}N_2-\sqrt{2}F)l\sqrt{2}(-\sqrt{2}) + (-\sqrt{2}N_2)l\sqrt{2}(-\sqrt{2})\Big] = \delta$$
$$u_D = \frac{\partial W}{\partial F_{Dx}} = \sum_{i=1}^{6} \frac{N_i l_i}{E_i S_i}\frac{\partial N_i}{\partial F_{Dx}} = \frac{1}{ES}\Big[(N_2+F-F_{Dx})l(-1)+0+0+0+0+0\Big] = 0$$

$N_2+F-F_{Dx}+N_2+F+N_2+F+N_2+2\sqrt{2}N_2+2\sqrt{2}N_2 = \delta ES$
$N_2+F-F_{Dx} = 0$

$3N_2+4\sqrt{2}N_2+2F+2\sqrt{2}F = \delta ES$

$$N_2 = \frac{\delta ES - 2F(1+\sqrt{2})}{3+4\sqrt{2}}, \qquad F_{Dx} = \frac{\delta ES + F(1+2\sqrt{2})}{3+4\sqrt{2}}$$

Now we can obtain the values of the remaining forces $(N_1 \dots N_6)$ by substitution of the statical redundants into the equations derived above from the conditions of static equilibrium.

Distribution of stresses:

stress

At the bars with the same cross section area, loaded by constant normal forces in simple tension (compression), the stress in each of them is constant; the highest stress value occurs in the bar with maximum normal force and can be calculated using the following formula:

$$\sigma_{\max} = \frac{N_{\max}}{S} = \frac{\max\{N_1,N_2,N_3,N_4,N_5,N_6\}}{S}.$$

Check for the limit state of elasticity: limit state

Since the bars are made of the same material (with the same yield stress), the safety factor of the whole system can be calculated as follows:: $k_K = \frac{\sigma_K}{\sigma_{\max}}$.