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Analysis:

Objective: to judge the influence of the shear force on the deformation parameters in a certain point of the bar.

Classification of the bar: straight, with statically determinate bearing (one fixed support), loaded by an external concentrated force $\vec{F}$.

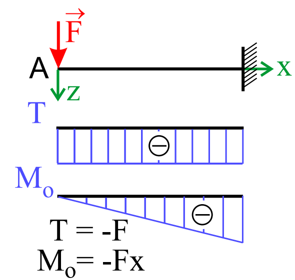
basic flexion

principal central axis

W_{M_o}

W_T

The distribution of inner resultants shows us that there are two non-zero components of the inner resultants in the bar, namely the bending moment $\vec{M}_o$ (in y-direction) and the shear force $\vec{T}$ (in z-direction). Thus the bar is loaded by basic flexion (the line of action of the bending moment is identical with a principal central axis of the cross section), and the shear force creates, moreover, loading by shear; thus the bar can be called beam. We comprehend the influence of the components of the inner resultants (bending moment and shear force) on the strain energy and calculate the contribution of both of them on the deflection and slope of the beam in the point A:



$$W = \int_0^l \frac{M_o^2}{2EJ_y} \mathrm{d}x + \frac{\beta}{2G} \int_0^l \frac{T^2(x)}{S(x)} \mathrm{d}x.$$

The displacement of the point of action A of the force $\vec{F}$ displacement

$$
\begin{aligned}
w &= w_{M_o} + w_T = \frac{\partial W_{M_o}}{\partial F} + \frac{\partial W_T}{\partial F} = \frac{1}{EJ}\int_0^l M_o \cdot \frac{\partial M_o}{\partial F}\mathrm{d}x + \frac{\beta}{G}\int_0^l \frac{T}{S} \cdot \frac{\partial T}{\partial F}\mathrm{d}x \\
&= \frac{1}{EJ}\int_0^l (-Fx)(-x)\mathrm{d}x + \frac{\beta}{G}\int_0^l \frac{-F}{S}(-1)\mathrm{d}x = \frac{Fl^3}{3EJ} + \beta\frac{Fl}{GS},
\end{aligned}
$$

J_y

G

β

where $J = \frac{\pi d^4}{64}$, $G = \frac{E}{2(1+\mu)}$, $S = \frac{\pi d^2}{4} = \frac{16J}{d^2}$, $\beta = \frac{32}{27}$.

The partial displacement of the point of action A created by

a) bending moment M_o

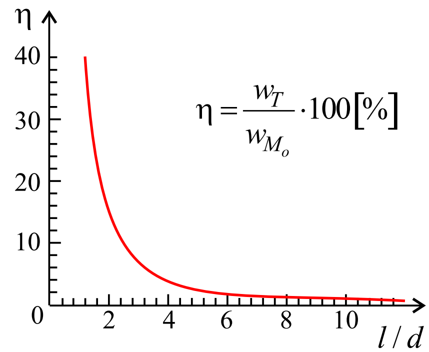
$$w_{M_o} = \frac{Fl^3}{3EJ}$$

b) shear force T

$$w_T = \beta\frac{Fl}{GS} = \frac{32}{27}Fl\frac{2(1+\mu)}{E}\frac{d^2}{16J} = 0,578\left(\frac{d}{l}\right)^2\frac{Fl^3}{3EJ}$$

The ratio of these partial displacements is determined by the relation

$$\frac{w_T}{w_{M_o}} = 0,578 \left(\frac{d}{l}\right)^2 .$$



The graph of the dependence of this quantity on the ratio $\frac{l}{d}$ shows us evidently that the influence of the shear force on the displacements of points of the centreline is **negligible** if the length dimensions of the bar are substantially higher than the transversal dimensions. For $l > 10d$ the shear force can be neglected.

Since the strain energy created by the shear force $\vec{T}$ is independent of the external load by the couple $\vec{\mathcal{M}}_d$, the value of derivative (and thus of the slope) will not be influenced by the shear force.