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Analysis:

Objectives: check of the limit state of elasticity..

limit state of elasticity

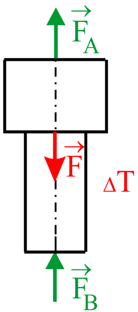
Classification of the bar: straight notched bar, loaded by isolated forces and temperature changes. The bar assumptions are not satisfied in the support and notch locations.

loading

bar assumptions

supports

Free body diagram:



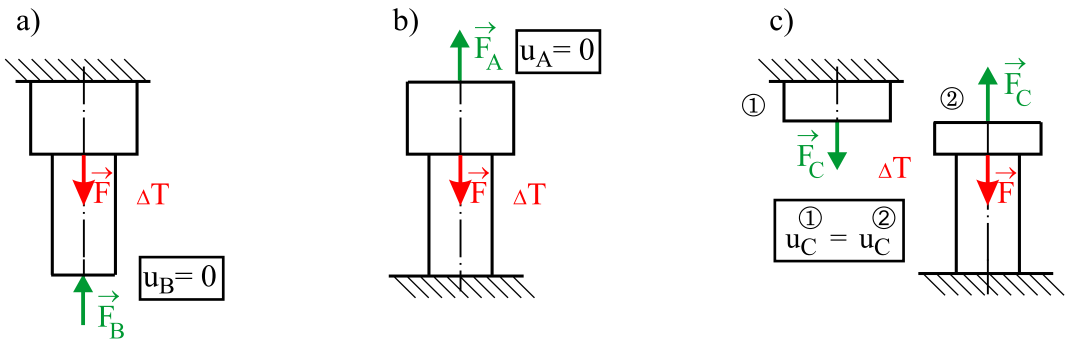
Resultants consisting of three force and three moment components can be created in each of the (fixed in 3D space) supports A and B. If we ensure a sufficient coaxiality and alignment of the supports, the components without any significant external loads in their direction will be negligible, and the only one significant component of the support resultant will be the force parallel to the bar centreline and thus also to the external loads.

Statical analysis:
a system of forces acting in the same line ⇒ $\nu = 1, \quad \mu = 2$
$s = \mu - \nu = 1 \quad \Rightarrow$ statically onefold indeterminate system
Applicable conditions of static equilibrium: $F_x: \quad F_A - F + F_B = 0$

statical analysis

Released structure:

released structure



Modification of the compatibility equations:

Compatibility equations must be expressed using the force and temperature loads. Hereto we can use

a) the formulas derived in the chapter „Expression of the deformation characteristic of the centreline“ posuv

$$u(x) = \int_0^x \frac{N(x)}{ES(x)} \mathrm{d}x.$$

As the bar is, moreover, the value of the temperature difference ΔT heated, it will show an additional elongation of $\Delta x_T = x\alpha\Delta T$.

Since we deal with a bar loaded by external forces and by temperature change ΔT, temperature change the displacement of a point of the centreline will be

$$u(x) = \int_0^x \frac{N(x)}{ES(x)} \mathrm{d}x + x\alpha\Delta T.$$

b) Castigliano's theorem.
The strain energy of a bar loaded in simple tension is

Castigliano's theorem

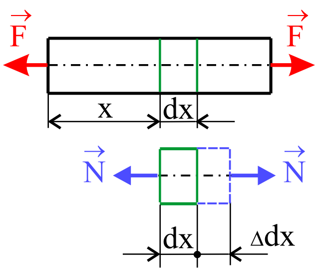
$$W = \int_0^l \frac{N^2(x)}{2ES(x)} \mathrm{d}x$$

Therefore the displacement u_K of the point of action of the force $\vec{F}_K$ (which acts upon the bar of the length l) in the direction of acting of this force is

strain energy

$$u_K = \frac{\partial W}{\partial F_K} = \int_0^l \frac{N(x)}{ES(x)} \frac{\partial N(x)}{\partial F_K} \mathrm{d}x.$$

We isolate a onefold elementary element (having the length of dx) from the bar as a free body; when the value of ΔT heated, this element's elongation is $\Delta \mathrm{d}x = \mathrm{d}x\alpha\Delta T$. An internal force $\vec{N}(x)$ acts upon this element and it does the work (by the elongation of the element $\Delta \mathrm{d}x$) equal to its strain energy W_T)

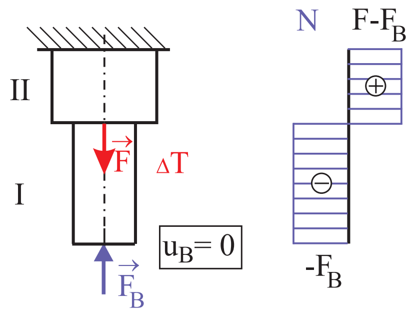


$$W_T(x) = N(x)\Delta \mathrm{d}x = N(x)\mathrm{d}x\alpha\Delta T.$$

The displacement of the point K of the centreline of the bar loaded by external forces and temperature change ΔT is:

$$u_K = \frac{\partial W_F}{\partial F_K} + \frac{\partial W_T}{\partial F_K} = \int_0^l \frac{N(x)}{ES(x)} \frac{\partial N(x)}{\partial F_K} \mathrm{d}x + \int_0^l \frac{\partial N(x)}{\partial F_K} \alpha \Delta T \mathrm{d}x.$$

We choose one of the released structures and create a compatibility equation, e.g. a):



Distribution of inner resultant:

$x \in (0, b): \quad N_I = -F_B$

$x \in (0, a): \quad N_{II} = F - F_B$

We express the compatibility equation in both of the above ways with the aim to test them and to be able to use the more advantageous of them in future solutions:

a) substitution into the derived formulas for deformation of the centreline:

$$u_B = \frac{-F_B}{ES_I} b + \frac{F - F_B}{ES_{II}} a + a\alpha\Delta T + b\alpha\Delta T = 0$$

b) Castigliano's theorem

$$u_B = \frac{\partial W_F}{\partial F_B} + \frac{\partial W_T}{\partial F_B} = \int_0^b \frac{-F_B}{ES_I} \cdot (-1) \mathrm{d}x + \int_0^a \frac{F - F_B}{ES_{II}} \cdot (-1) \mathrm{d}x + \int_0^a (-1) \cdot \alpha \Delta T \mathrm{d}x + \int_0^b (-1) \cdot \alpha \Delta T \mathrm{d}x = 0$$

Set of equations and their solution:

$$F_A - F + F_B = 0$$

$$\frac{-F_B}{ES_I}b + \frac{F - F_B}{ES_{II}}a + a\alpha\Delta T + b\alpha\Delta T = 0$$

$$F_B = \frac{(a+b)\alpha E\Delta T + \frac{Fa}{S_{II}}}{\frac{a}{S_{II}} + \frac{b}{S_I}}, \qquad F_A = F - F_B = \frac{\frac{Fb}{S_I} - (a+b)\alpha E\Delta T}{\frac{a}{S_{II}} + \frac{b}{S_I}}.$$

Discussion:

If a state can occur in operation of a statically indeterminate supported body (e.g. in relation to the assemblage of the bars), in which the bar is loaded only by the temperature change, this state should be checked as well (by substitution $F = 0$ into these relations). The stress state can be worse in some points of the bar under these conditions than when the force is acting.

a) state after assemblage:

$$F = 0 \quad \Rightarrow \quad F_B = \frac{(a+b)\alpha E\Delta T}{\frac{a}{S_{II}} + \frac{b}{S_I}},$$

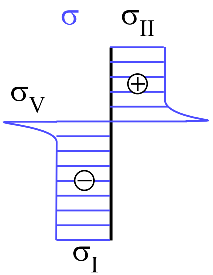
b) state after assemblage and loaded by the force $\vec{F}$:

$$F_B = \frac{(a+b)\alpha E\Delta T + \frac{Fa}{S_{II}}}{\frac{a}{S_{II}} + \frac{b}{S_I}}.$$

Distribution of stresses:

stress

notch



Since the bar is loaded in simple tension (compression) (except for the notch surroundings), the stress is constant across the section and can be calculated using the following formula:

$$\sigma(x) = \frac{N(x)}{S(x)}: \qquad \sigma_I = \frac{N_I}{S_I} = \frac{F_B}{S_I}, \quad \sigma_{II} = \frac{N_{II}}{S_{II}} = \frac{F_A}{S_{II}}$$

Extreme stress in the notch location:

$$\sigma_V = \alpha_V \sigma_{nom} = \alpha_V \sigma_I = \alpha_V \frac{F_B}{S_I}$$

Check for the limit state of elasticity:

limit state

We determine the section with extreme stress (dangerous section) from the stress distribution and calculate the safety factor against the limit state of elasticity:

$$k_K = \frac{\sigma_K}{\sigma_{\max}}.$$

Note:

We do not check the welds in the supports A and B, the results depend, in addition to the notch effect, on the technology of welding.