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Analysis:

A system of three straight slender bars loaded in their joint, each of the bars supported by the base, all the supports modelled as pin supports.

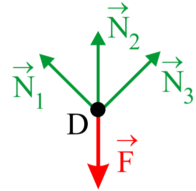
soustava prutů

Free body diagram:

We isolate the joint D as a free body:

Statical analysis: $\mu = 3$, $\nu = 2$ (central in-plane system of forces (intersecting each other in one point))

$s = \mu - \nu = 1 \quad \Rightarrow$ statically onefold indeterminate problem



analysis

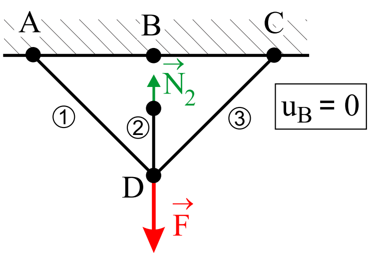
released structure

Released structure:

The easiest way of how to create the primary (released) structure is usually to release a joint between one of the bars and the base; the deformation of the base is negligible so that the compatibility equation has a simpler initial form.

For instance, we release the bar No. 2 in the point B and introduce the force N_2 (statical redundant) here; the corresponding compatibility equation is $u_B = 0$.

$$u_B = \frac{\partial W}{\partial N_2} = \sum_{i=1}^{3} \frac{N_i l_i}{ES} \frac{\partial N_i}{\partial N_2} = 0$$



Castigliagno's theorem

Applicable conditions of static equilibrium: $\sum F_x = 0: \quad -N_1 \sin\alpha + N_3 \sin\alpha = 0$ static equilibrium
$\sum F_y = 0: \quad N_1 \cos\alpha + N_2 + N_3 \cos\alpha - F = 0$
We express the **forces in the bars** using the statical redundant (chosen by the form of the released structure): applicable conditions

$$N_1 = N_3, \qquad N_1 = \frac{F - N_2}{2\cos\alpha}.$$

Modification of the compatibility equation and calculation of the forces in the bars:

$$u_B = \frac{1}{ES}\left[\frac{(F-N_2)l_1}{2\cos\alpha}\left(-\frac{1}{2\cos\alpha}\right)\cdot 2 + N_2 l_2 \cdot 1\right] = \frac{1}{ES}\left[\frac{(N_2 - F)l}{2\cos^3\alpha} + N_2 l\right] = 0$$

$$N_2 = \frac{F}{2\cos^3\alpha + 1}, \qquad N_1 = N_3 = \frac{F - N_2}{2\cos\alpha}$$

Distribution of stresses: stress

Since each of the bars (of the same cross section) is loaded in simple tension (compression) by the normal force constant along the bar centreline, the stress is constant in each of them and can be calculated using the following formula:

$$\sigma(x) = \frac{N(x)}{S(x)}: \qquad \sigma_1 = \sigma_3 = \frac{N_1}{S}, \quad \sigma_2 = \frac{N_2}{S}.$$

Check for the limit state of elasticity:

limit state

Since the stress is constant in each of the bars and they are made of the same material (with the same yield stress), we can calculate the safety factor against the limit state of elasticity valid for all the system by substituting the maximum of the calculated stress values into the formula:

$$k_K = \frac{\sigma_K}{\sigma_{\max}} \, .$$

Note:

the calculation does not account for the technology of the joints that is decissive for the choice of the acceptable safety factor value.