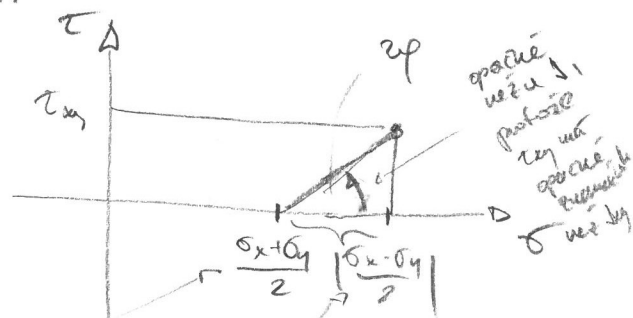
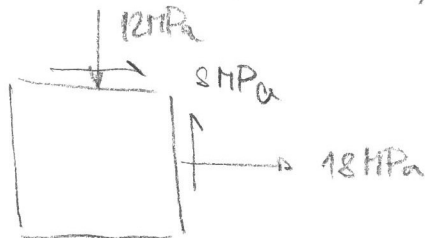


Pi. Najdete hl. hodnoty a směr tenzoru napětí



$$\sigma_x = 18 \text{ MPa}$$

$$\tau_{xy} = 8 \text{ MPa}$$

$$\sigma_y = -12 \text{ MPa}$$

$$\frac{\sigma_x + \sigma_y}{2} = \frac{18 + (-12)}{2} = 3 \text{ MPa}$$

$$\left| \frac{\sigma_x - \sigma_y}{2} \right| = \frac{18 - (-12)}{2} = 15 \text{ MPa}$$

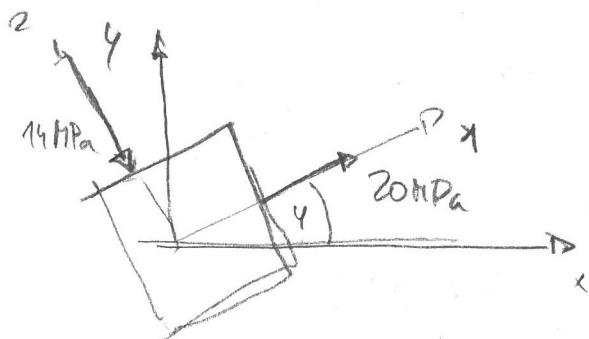
$$\tan 2\varphi = \frac{2\tau_{xy}}{|\sigma_x - \sigma_y|} = \frac{8}{15} = 0.533$$

$$2\varphi = 28.07^\circ \Rightarrow \varphi = 14.04^\circ$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{15^2 + 8^2} = 17 \text{ MPa}$$

$$\sigma_{\min, \max} = \frac{\sigma_x + \sigma_y}{2} \pm R = 3 \pm 17 \Rightarrow \sigma_{\min} = -14 \text{ MPa}$$

$$\sigma_{\max} = 20 \text{ MPa}$$



Alternativa

$$I_1 = \text{tr}(\sigma) = \sigma_x + \sigma_y$$

$$I_2 = \begin{vmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{vmatrix}$$

$$I_3 = 0$$

$$\sigma = \begin{bmatrix} 18 & 8 \\ 8 & -12 \end{bmatrix}$$

$$\Rightarrow I_1 = 6 \text{ MPa}$$

$$I_2 = -18 \cdot 12 - 8^2 = -280$$

$$\begin{vmatrix} 18 - \sigma_i & 8 \\ 8 & -12 - \sigma_i \end{vmatrix} = (18 - \sigma_i)(-12 - \sigma_i) - 8^2 = 0$$

$$18(-12) + 12\sigma_i - 18\sigma_i + \sigma_i^2 - 8^2 = 0$$

$$\sigma_i^2 + (-18 + 12)\sigma_i + 18(-12) - 8^2 = 0$$

$$\sigma_i^2 - I_1 \sigma_i + I_2 = 0$$

$$\sigma_i^2 - 6\sigma_i - 280 = 0 \Rightarrow \sigma_{1,2} =$$

$$\frac{6 + \sqrt{6^2 - 4(-280)}}{2} = 20 \text{ MPa}$$

$$\frac{6 - \sqrt{6^2 - 4(-280)}}{2} = -14 \text{ MPa}$$

n₁:

$$\begin{bmatrix} 18-20 & 8 \\ 8 & -12-20 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$\begin{bmatrix} -2 & 8 \\ 8 & -32 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$\left. \begin{array}{l} \text{Zurück } y=1 \\ -2x+8=0 \\ x=4 \end{array} \right\} \Rightarrow$$

$$\underline{n}_1 = \begin{bmatrix} \frac{4}{\sqrt{4^2+1^2}} \\ \frac{1}{\sqrt{4^2+1^2}} \end{bmatrix} = \begin{bmatrix} 0.97 \\ 0.24 \end{bmatrix} = \begin{bmatrix} \cos 14.04 \\ \sin 14.04 \end{bmatrix}$$

n₂:

$$\begin{bmatrix} 18+14 & 8 \\ 8 & -12+14 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$\begin{bmatrix} 32 & 8 \\ 8 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$\text{Zurück } y=1$$

$$8x+2=0 \Rightarrow x = -\frac{1}{4}$$

$$\left. \begin{array}{l} \Rightarrow \underline{n}_2 = \begin{bmatrix} \frac{-\frac{1}{4}}{\sqrt{(\frac{1}{4})^2+1}} \\ \frac{1}{\sqrt{(\frac{1}{4})^2+1}} \end{bmatrix} = \begin{bmatrix} -0.24 \\ 0.97 \end{bmatrix} \\ = \begin{bmatrix} -\sin 14.04 \\ \cos 14.04 \end{bmatrix} \end{array} \right\}$$