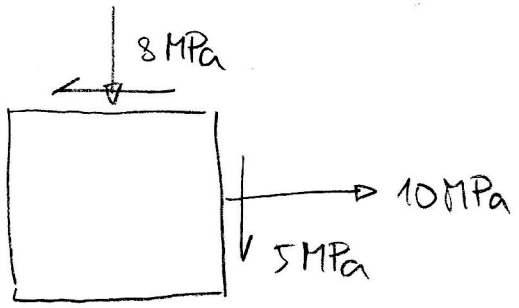
Př: Najděte hl. hodnoty a směry tenzoru napětí.



$$T_\sigma = \begin{bmatrix} 10 & -5 \\ -5 & -8 \end{bmatrix}$$

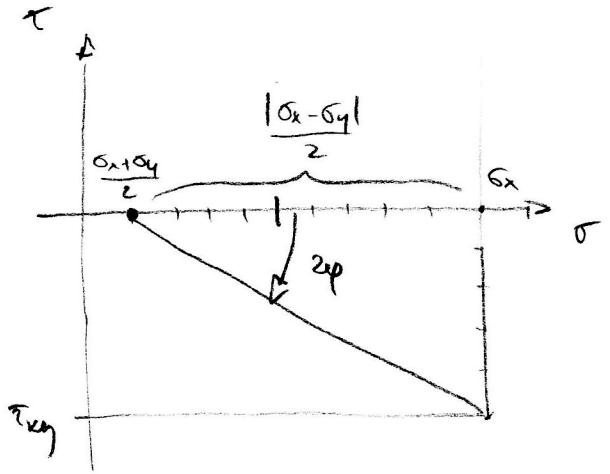
$\sigma_x = 10\,MPa$

$\sigma_y = -8\,MPa$

$\tau_{xy} = -5\,MPa$

$$\frac{\sigma_x+\sigma_y}{2} = \frac{10-8}{2} = 1\,MPa$$

$$\left|\frac{\sigma_x-\sigma_y}{2}\right| = \frac{10+8}{2} = 9\,MPa$$

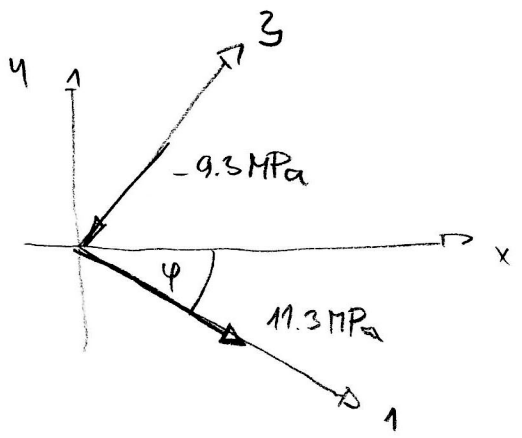


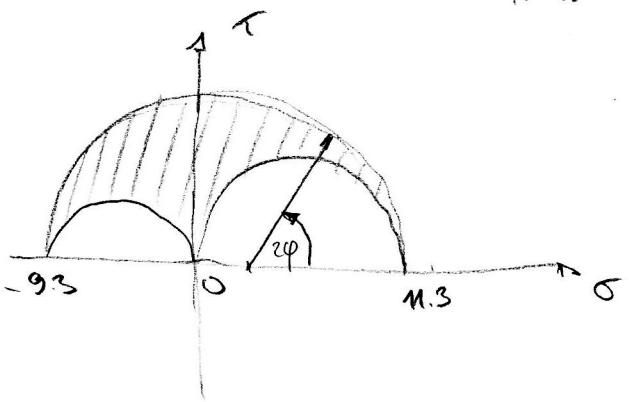
$$\operatorname{tg} 2\varphi = \frac{|\tau_{xy}|}{\frac{|\sigma_x-\sigma_y|}{2}} = \frac{2\cdot 5}{18} \Rightarrow$$

$$\Rightarrow \varphi = (\operatorname{arctan}\tfrac{10}{18})\cdot\tfrac{1}{2} = 14.5^\circ$$

$$R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{9^2+5^2} = 10.3\,MPa$$

$$\sigma_{max,min} = \frac{\sigma_x+\sigma_y}{2} \pm R = 1 \pm 10.3 = \begin{cases} 11.3\,MPa \\ -9.3 \end{cases}$$





Alternativa I:

$$I_1 = tr(T_\sigma) = \sigma_x + \sigma_y = 10 - 8 = 2\ MPa$$

$$I_2 = \begin{vmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{vmatrix} = \begin{vmatrix} 10 & -5 \\ -5 & -8 \end{vmatrix} = -80 - 25 = -105\ MPa^2$$

$$I_3 = 0$$

$$\sigma_i^3 - I_1\sigma_i^2 + I_2\sigma_i - I_3 = 0$$

$$\sigma_i(\sigma_i^2 - 2\sigma_i + (-105)) = 0$$

$$\sigma_i = \begin{cases} \dfrac{2 + \sqrt{4 + 4\cdot(105)}}{2} = 11.3\ MPa \\ 0 \\ \dfrac{2 - \sqrt{4 + 4\cdot 105}}{2} = -9.3\ MPa \end{cases}$$

$$\Rightarrow (10 - \sigma_i)(-8 - \sigma_i) - 25 = 0$$

$$\sigma_i^2 + (-10 + 8)\sigma_i - 80 - 25 = 0$$

$$\sigma_i^2 - 2\sigma_i - 105 = 0$$

$$\sigma_i = \begin{cases} 11.3\ MPa \\ \cdots\cdots\ 0\ MPa \\ -9.3\ MPa \end{cases}$$

$$\begin{bmatrix} 10 - 11.3 & -5 \\ -5 & -8 - 11.3 \end{bmatrix} \begin{bmatrix} n_x^1 \\ n_x^2 \end{bmatrix} = 0$$

$$\begin{bmatrix} -1.3 & -5 \\ -5 & -19.3 \end{bmatrix} \begin{bmatrix} n_x^1 \\ n_x^2 \end{bmatrix} = 0$$

$$n_x^1 = 1 \Rightarrow \begin{matrix} -1.3 - 5n_x^2 = 0 \\ -5 - 19.3n_x^2 = 0 \end{matrix} \Rightarrow n_x^2 = -\frac{1.3}{5} = -\frac{5}{19.3} = -0.26$$

Alternativa II (varianta aceasta):

$$\begin{bmatrix} \sigma_x - \sigma_i & \tau_{xy} \\ \tau_{xy} & \sigma_y - \sigma_i \end{bmatrix} \begin{bmatrix} n_x^i \\ n_y^i \end{bmatrix} = 0$$

$$\begin{bmatrix} 10 - \sigma_i & -5 \\ -5 & -8 - \sigma_i \end{bmatrix} \begin{bmatrix} n_x^i \\ n_y^i \end{bmatrix} = 0$$

$$\det\left(\begin{bmatrix} 10 - \sigma_i & -5 \\ -5 & -8 - \sigma_i \end{bmatrix}\right) = 0 \Rightarrow$$

$$\underline{n}_1 = \begin{bmatrix} 1 \\ -0.26 \end{bmatrix} \Rightarrow \underline{n}_1 = \frac{1}{\sqrt{1 + 0.26^2}} \cdot \begin{bmatrix} 1 \\ -0.26 \end{bmatrix} = \begin{bmatrix} 0.97 \\ -0.25 \end{bmatrix}$$

$$\underline{n}_3 = (0.25, 0.97) = \begin{bmatrix} 0.25 \\ 0.97 \end{bmatrix}$$