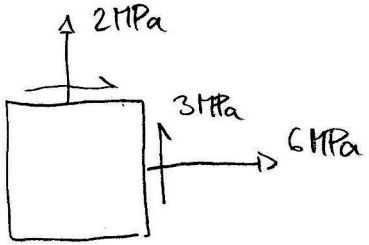
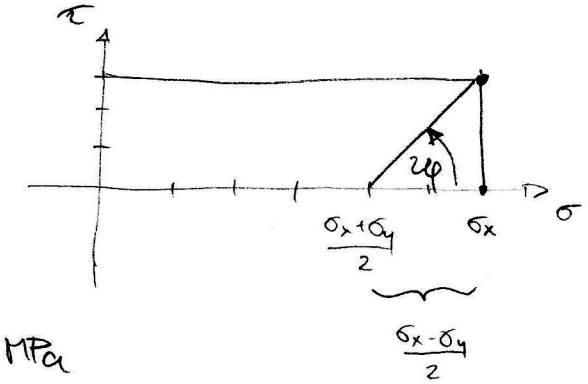
Př: Najděte hl. napětí a jejich směry.



$$\left.\begin{aligned}\sigma_x &= 6\,\text{MPa}\\ \sigma_y &= 2\,\text{MPa}\\ \tau_{xy} &= 3\,\text{MPa}\end{aligned}\right\} \Rightarrow \overline{\overline{T}}_\sigma = \begin{bmatrix} 6 & 3 \\ 3 & 2 \end{bmatrix}$$

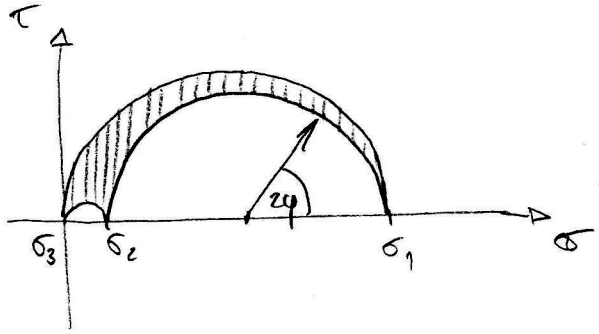
Varianta O:

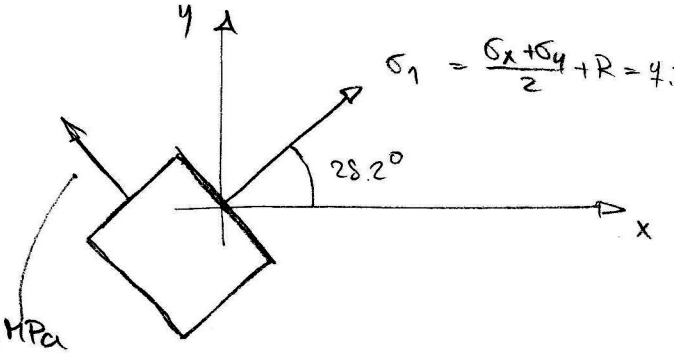
$$\left.\begin{aligned}\frac{\sigma_x+\sigma_y}{2} &= \frac{6+2}{2} = 4\,\text{MPa}\\ \frac{|\sigma_x-\sigma_y|}{2} &= \frac{6-2}{2} = 2\,\text{MPa}\end{aligned}\right\} \Rightarrow$$



$$2\varphi = \arctan\frac{|2\tau_{xy}|}{|\sigma_x-\sigma_y|} = \frac{2\cdot 3}{4} = 1{,}5 \Rightarrow \varphi = 28.2°$$

$$R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{2^2+3^2} = 3.6\,\text{MPa}$$





$$\sigma_1 = \frac{\sigma_x+\sigma_y}{2} + R = 7.6\,\text{MPa}$$

$$\sigma_2 = \frac{\sigma_x+\sigma_y}{2} - R = 0.4\,\text{MPa}$$

Varianta I:

$$I_1 = \operatorname{tr}(\overline{\overline{T}}_\sigma) = \sigma_x+\sigma_y = 6+2 = 8\,\text{MPa}$$

$$I_2 = \begin{vmatrix}\sigma_x & \tau_{xy}\\ \tau_{xy} & \sigma_y\end{vmatrix} = \begin{vmatrix} 6 & 3 \\ 3 & 2\end{vmatrix} = 12-9 = 3\,\text{MPa}^2$$

$$I_3 = 0$$

$$\Rightarrow \sigma_i^3 - I_1\sigma_i^2 + I_2\sigma_i - I_3 = 0$$

$$\sigma_i(\sigma_i^2 - 8\sigma_i + 3) = 0$$

$$\frac{8+\sqrt{8^2-4\cdot 3}}{2} = 7.6\,\text{MPa}$$

$$\frac{8-\sqrt{8^2-4\cdot 3}}{2} = 0.4\,\text{MPa}$$

$$0\,\text{MPa}$$

Varianta II:

$$\begin{bmatrix} \sigma_x-\sigma_i & \tau_{xy} \\ \tau_{xy} & \sigma_y-\sigma_i \end{bmatrix}\begin{bmatrix} n_x^i \\ n_y^i \end{bmatrix}=0 \Rightarrow \begin{bmatrix} 6-\sigma_i & 3 \\ 3 & 2-\sigma_i \end{bmatrix}\begin{bmatrix} n_x^i \\ n_y^i \end{bmatrix}=0 \Rightarrow$$

$$\begin{vmatrix} 6-\sigma_i & 3 \\ 3 & 2-\sigma_i \end{vmatrix}=0 \Rightarrow (6-\sigma_i)(2-\sigma_i)-9=0 \Rightarrow$$

$$\Rightarrow \sigma_i^2+(-2-6)\sigma_i+(12-9)=0 \Rightarrow \sigma_i^2-8\sigma_i+3=0$$

$$\sigma_{1,2}=\begin{cases} 7.6\ \text{MPa} \\ 0.4\ \text{MPa} \end{cases}$$

$$\begin{bmatrix} 6-7.6 & 3 \\ 3 & 2-7.6 \end{bmatrix}\begin{bmatrix} n_x^1 \\ n_y^1 \end{bmatrix}=0 \Rightarrow \begin{bmatrix} -1.6 & 3 \\ 3 & -5.6 \end{bmatrix}\begin{bmatrix} n_x^1 \\ n_y^1 \end{bmatrix}=0$$

Zvolíme $n_x^1=1$:

$$\begin{bmatrix} -1.6 & 3 \\ 3 & -5.6 \end{bmatrix}\begin{bmatrix} 1 \\ n_y^1 \end{bmatrix}=0 \Rightarrow \left.\begin{matrix} -1.6+3n_y^1=0 \\ 3-5.6n_y^1=0 \end{matrix}\right\} \Rightarrow n_y^1=\frac{1.6}{3}=\frac{3}{5.6}=0.54$$

$$\underline{n}^1=\begin{bmatrix} 1 \\ 0.54 \end{bmatrix} \equiv \frac{1}{\sqrt{1+0.54^2}}\cdot\begin{bmatrix} 1 \\ 0.54 \end{bmatrix}=\begin{bmatrix} 0.88 \\ 0.48 \end{bmatrix}, \quad \underline{n}^2=\begin{bmatrix} -0.48 \\ 0.88 \end{bmatrix}$$