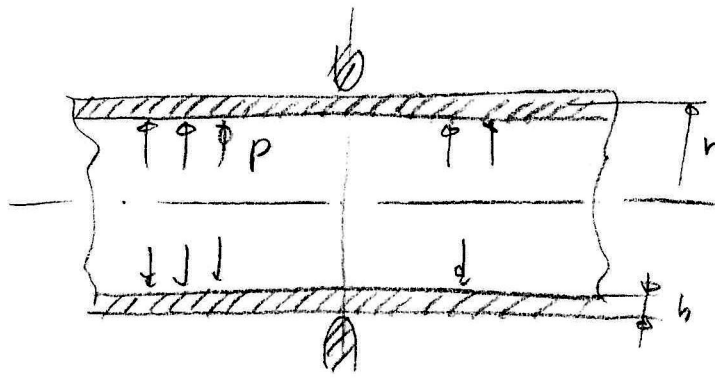


Pr:



k HSpurčnost: učebe bezpečnost

mat.: $\nu = 0.3$

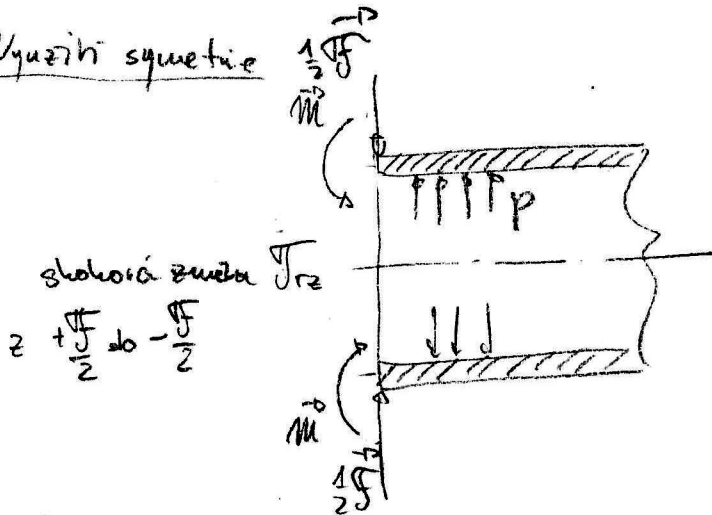
$E = 2.1 \cdot 10^5 \text{ MPa}$

geom.: $r = 20 \text{ mm}$

$h = 1 \text{ mm}$

zatížení: $p = 1 \text{ MPa}$

1. Využití symetrie



2. Okrajové podmínky a deformace

$\sigma_{rz}|_{z=0} = \frac{1}{2} p$ podmínky

$\tau_{rz}|_{z=0} = 0$

$u|_{z=0} = 0$

$v|_{z=0} = 0$

3. Počítání řešení

$$u_p = \frac{r^2}{E} \left[p - \frac{\nu}{r} \left(C_0 - \int p \, dz \right) \right] = \frac{r^2}{E} \left[p - \frac{\nu}{r} C_0 \right] = \frac{20^2}{2.1 \cdot 10^5} \left[1 - \frac{0.3}{20} C_0 \right] = 1.9 \cdot 10^{-3} - 2.86 \cdot 10^{-5} C_0$$

$$\beta = \sqrt[4]{\frac{3(1-\nu^2)}{r^2 h^2}} = \sqrt[4]{\frac{3(1-0.3^2)}{20^2 \cdot 1^2}} = 2.84 \cdot 10^{-1} \frac{1}{\text{mm}}, \quad B = \frac{E h^3}{12(1-\nu^2)} = \frac{2.1 \cdot 10^5 \cdot 1^3}{12(1-0.3^2)} = 1.92 \cdot 10^4 \text{ N} \cdot \text{mm}$$

$$u = u_p + \tilde{u} = 1.9 \cdot 10^{-3} - 2.86 \cdot 10^{-5} C_0 + e^{-2.84 \cdot 10^{-1} z} (C_1 \sin 2.84 \cdot 10^{-1} z + C_2 \cos 2.84 \cdot 10^{-1} z)$$

$$v = -2.84 \cdot 10^{-1} e^{-2.84 \cdot 10^{-1} z} [C_1 (\sin 2.84 \cdot 10^{-1} z - \cos 2.84 \cdot 10^{-1} z) + C_2 (\sin 2.84 \cdot 10^{-1} z + \cos 2.84 \cdot 10^{-1} z)]$$

$$w = \frac{1-\nu^2}{E} \int \frac{C_0}{h} \, dz - \frac{\nu}{r} \int u \, dz = \frac{1-0.3^2}{2.1 \cdot 10^5} \int \frac{C_0}{1} \, dz - \frac{0.3}{20} \int [1.9 \cdot 10^{-3} - 2.86 \cdot 10^{-5} C_0] \, dz$$

$$= \frac{0.3}{20} \left[\left(-\frac{1}{2\beta} \right) e^{-\beta z} ((C_1 + C_2) \cos \beta z + (C_1 - C_2) \sin \beta z) \right] = 4.33 \cdot 10^{-6} C_0 z -$$

$$- 2.85 \cdot 10^{-5} z + 4.29 \cdot 10^{-4} C_0 z + \frac{0.3}{20 \cdot 2.84 \cdot 10^{-1}} e^{-2.84 \cdot 10^{-1} z} [(C_1 + C_2) \cos 2.84 \cdot 10^{-1} z + (C_1 - C_2) \sin 2.84 \cdot 10^{-1} z] = (-2.85 \cdot 10^{-5} + 4.46 \cdot 10^{-6} C_0) z + 2.61 \cdot 10^{-2} e^{-2.84 \cdot 10^{-1} z} \cdot$$

$$\cdot [(C_1 + C_2) \cos 2.84 \cdot 10^{-1} z + (C_1 - C_2) \sin 2.84 \cdot 10^{-1} z]$$

$$\sigma_{rz} = -2 \cdot 1.92 \cdot 10^4 \cdot (2.84 \cdot 10^{-1})^3 e^{-2.84 \cdot 10^{-1} z} (C_1 (\sin 2.84 \cdot 10^{-1} z + \cos 2.84 \cdot 10^{-1} z) + C_2 (-\sin 2.84 \cdot 10^{-1} z + \cos 2.84 \cdot 10^{-1} z)) = -9.08 \cdot 10^2 e^{-2.84 \cdot 10^{-1} z} [C_1 (\sin 2.84 \cdot 10^{-1} z + \cos 2.84 \cdot 10^{-1} z) + C_2 (-\sin 2.84 \cdot 10^{-1} z + \cos 2.84 \cdot 10^{-1} z)]$$

$$M_z = -2.192 \cdot 10^4 \cdot (2.84 \cdot 10^{-1})^2 e^{-2.84 \cdot 10^{-1} z} (-C_1 \cos 2.84 \cdot 10^{-1} z + C_2 \sin 2.84 \cdot 10^{-1} z) = -3.16 \cdot 10^3 \cdot e^{-2.84 \cdot 10^{-1} z} \cdot (-C_1 \cos 2.84 \cdot 10^{-1} z + C_2 \sin 2.84 \cdot 10^{-1} z)$$

4. Virenen $C_0, C_1, C_2, \bar{F}$ a M

$$\bar{F}_z \Big|_{z=0} = -9.08 \cdot 10^2 [C_1 + C_2] = \frac{1}{2} \bar{F}$$

$$v \Big|_{z=0} = -C_1 + C_2 = 0$$

$$M_z \Big|_{z=0} = C_0 = 0$$

$$u \Big|_{z=0} = 1.9 \cdot 10^{-3} - 2.86 \cdot 10^{-5} C_0 + C_2 = 0$$

$$-C_1 + C_2 = 0$$

$$C_0 = 0$$

$$-(9.08 \cdot 10^2 C_1 + 9.08 \cdot 10^2 C_2) - 0.5 \bar{F} = 0$$

$$-2.86 \cdot 10^{-5} C_0 + C_2 = -1.9 \cdot 10^{-3}$$

$$C_0 = 0$$

$$C_2 = -1.9 \cdot 10^{-3}$$

$$C_1 = C_2 = -1.9 \cdot 10^{-3}$$

$$\bar{F} = -2 \cdot (9.08 \cdot 10^2 \cdot (-1.9 \cdot 10^{-3}) + 9.08 \cdot 10^2 (-1.9 \cdot 10^{-3})) = +6.9 \cdot 10^0 \frac{N}{mm}$$

5. Peseur

$$u = 1.9 \cdot 10^{-3} - 1.9 \cdot 10^{-3} e^{-2.84 \cdot 10^{-1} z} (\sin 2.84 \cdot 10^{-1} z + \cos 2.84 \cdot 10^{-1} z)$$

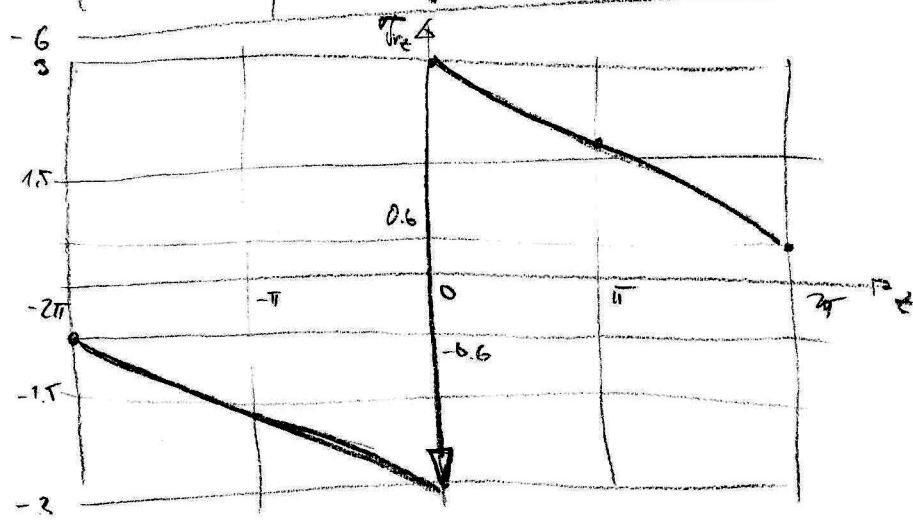
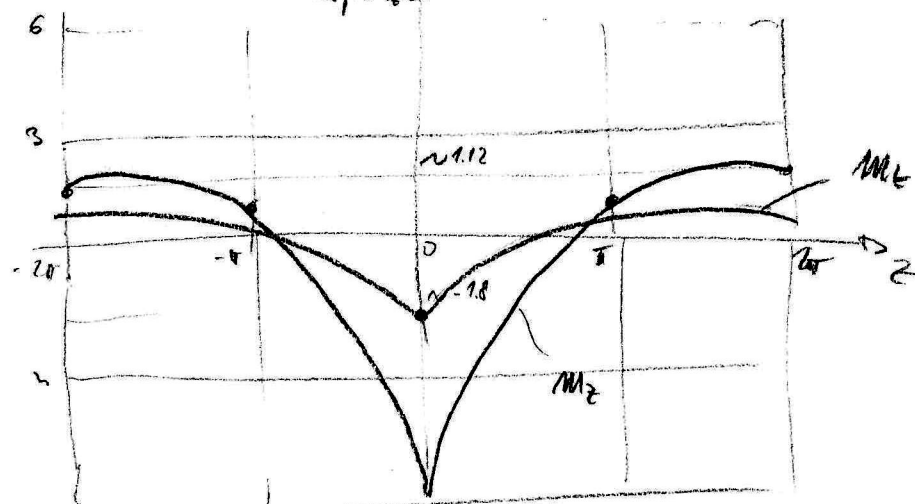
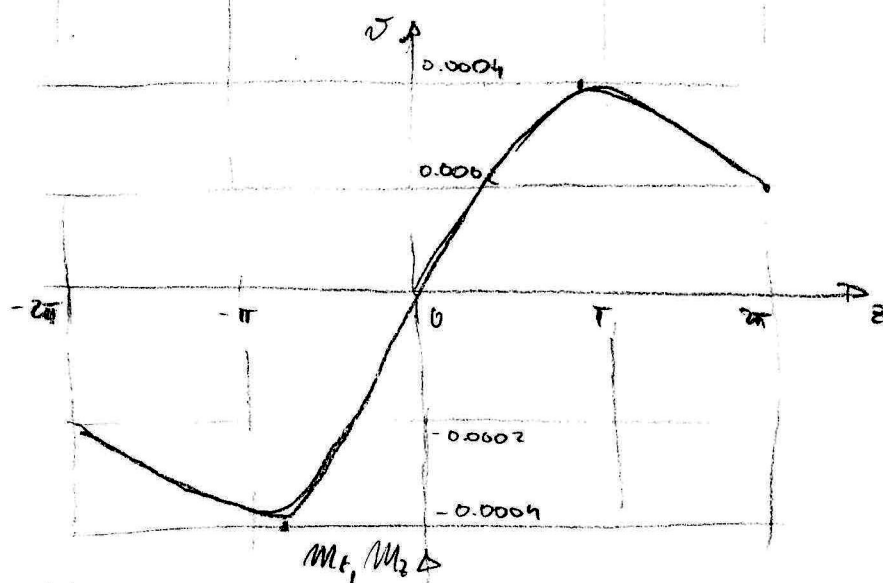
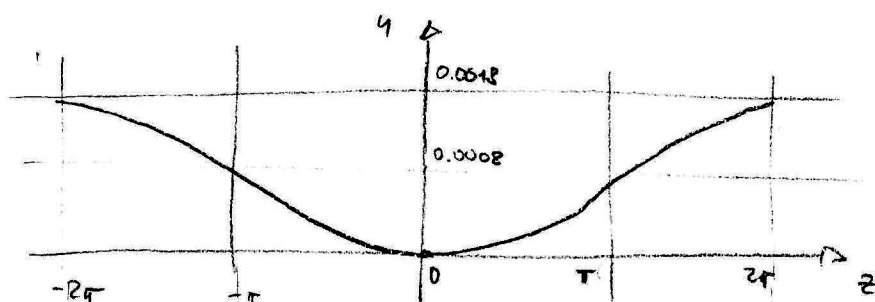
$$M_z = 0$$

$$\begin{aligned} M_z &= r \left(p + \frac{d\bar{F}_z}{dz} \right) = 20 \cdot \left(1 + 9.08 \cdot 10^2 (-2.84 \cdot 10^{-1}) e^{-2.84 \cdot 10^{-1} z} \cdot (-1.9 \cdot 10^{-3}) [\sin 2.84 \cdot 10^{-1} z + \cos 2.84 \cdot 10^{-1} z] \right. \\ &\quad \left. + 9.08 \cdot 10^2 e^{-2.84 \cdot 10^{-1} z} (-1.9 \cdot 10^{-3}) \cdot 2.84 \cdot 10^{-1} \cdot (\cos 2.84 \cdot 10^{-1} z - \sin 2.84 \cdot 10^{-1} z - \cos 2.84 \cdot 10^{-1} z - \sin 2.84 \cdot 10^{-1} z) \right) \\ &= 20 \left(1 + 4.95 \cdot 10^{-1} e^{-2.84 \cdot 10^{-1} z} (2 \cdot \cos 2.84 \cdot 10^{-1} z + 2 \sin 2.84 \cdot 10^{-1} z) \right) \end{aligned}$$

$$M_z = -3.16 \cdot 10^3 e^{-2.14 \cdot 10^{-1} z} \left(+1.9 \cdot 10^{-3} \cos 2.84 \cdot 10^{-1} z - 1.9 \cdot 10^{-3} \sin 2.84 \cdot 10^{-1} z \right) =$$

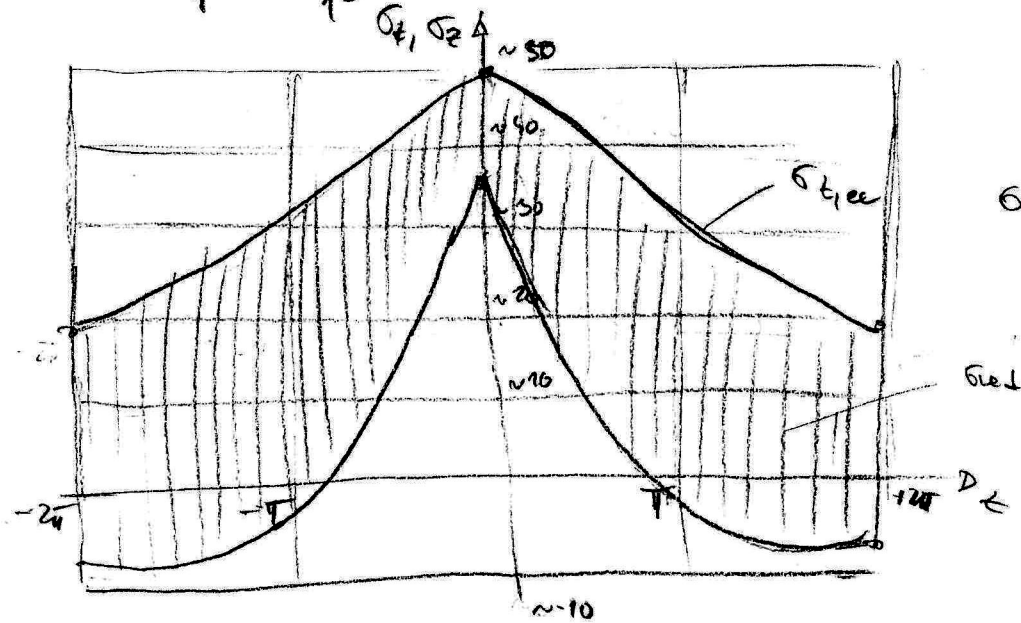
$$= 6 \cdot e^{-2.14 \cdot 10^{-1} z} \left(-\cos 2.84 \cdot 10^{-1} z + \sin 2.84 \cdot 10^{-1} z \right)$$

$$M_t = 0.3 \cdot M_z = 1.8 e^{-2.14 \cdot 10^{-1} z} \left(-\cos 2.84 \cdot 10^{-1} z + \sin 2.84 \cdot 10^{-1} z \right)$$



$$\sigma_{z,ae} = \pm M_z \cdot \frac{c}{I_z} = -6 M_z$$

$$\sigma_{t,ae} = \frac{M_t}{I} \pm \frac{6 M_t}{I^2} = M_t - 6 M_t$$



$$\sigma_{red} = \left| \sigma_{t,ae} - \sigma_{z,ae} \right|_{z=?}$$