

Pr: Určete bezpečnost k HS pružnosti u soustavy desek podle obr.

mat: $\nu = 0.3$

$E = 2.1 \cdot 10^5 \text{ MPa}$

geom: $r_1 = 20 \text{ mm}$

$r_2 = 40, r_3 = 60 \text{ mm}$

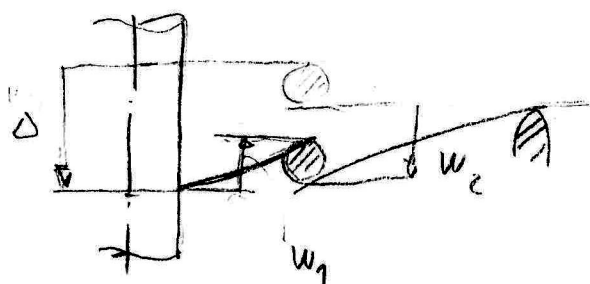
$h = 1$

ztláčen: $\Delta = 3 \cdot h = 3 \text{ mm}$

$B_1 = \frac{2.1 \cdot 10^5 \cdot 1^3}{12(1-0.3^2)} = 1.92 \cdot 10^4 \text{ N/mm}$

$B_2 = \frac{2.1 \cdot 10^5 \cdot 2^3}{12(1-0.3^2)} = 1.54 \cdot 10^5 \text{ N/mm}$

1. Volurní a deformační podmínky



$|w_1| + |w_2| = \Delta$

2. Okrajové podmínky

$\sigma_1|_{r=r_1} = 0$

$M_{\sigma_2}|_{r=r_2} = 0$

$|w_1| + |w_2| = \Delta$

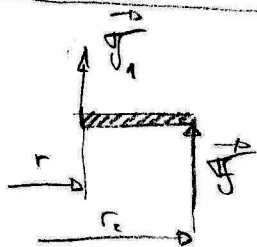
$w_1|_{r=r_1} = 0$

$M_{\sigma_2}|_{r=r_2} = 0$

$M_{\sigma_1}|_{r=r_2} = 0$

$w_2|_{r=r_3} = 0$

3. Pochybné řešení

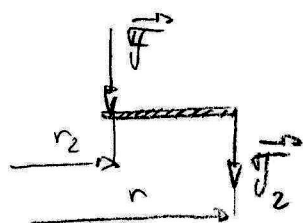


$\sigma_1 r = -\sigma_2 r$

$\sigma_1 = -\frac{r_2}{r} \sigma_2 = -\frac{40}{r} \sigma_2$

$\sigma_{p1} = -\frac{1}{r B_1} \int r \sigma_1 dr dr = -\frac{1}{r \cdot 1.92 \cdot 10^4} \cdot \int r \left(-\frac{40 \sigma_2}{r} \right) dr dr = \frac{2.08 \cdot 10^{-4}}{r} \int r \sigma_2 dr$

$\sigma_2 \ln r dr = \frac{2.08 \cdot 10^{-4}}{r} \sigma_2 \cdot \frac{1}{2} r^2 \left(\ln r - \frac{1}{2} \right) = 1.04 \cdot 10^{-4} \sigma_2 r (\ln r - 0.5)$



$\sigma_2 r = -\sigma_1 r$

$\sigma_2 = -\frac{r_2}{r} \sigma_1 = -\frac{40}{r} \sigma_1$

$\sigma_{p2} = -\frac{1}{r B_2} \int r \sigma_2 dr dr = -\frac{1}{r \cdot 1.54 \cdot 10^5} \cdot \int r \left(-\frac{40 \sigma_1}{r} \right) dr dr = 1.3 \cdot 10^{-4} \sigma_1 r (\ln r - 0.5)$

4. Уточнені моменти, радіус

$$\sigma_1 = c_{11}r + \frac{c_{12}}{r} + 1.04 \cdot 10^{-4} F \cdot r (\ln r - 0.5)$$

$$\frac{d\sigma_1}{dr} = c_{11} - \frac{c_{12}}{r^2} + 1.04 \cdot 10^{-4} F (\ln r - 0.5) + 1.04 \cdot 10^{-4} F \cdot r \cdot \frac{1}{r} = c_{11} - \frac{c_{12}}{r^2} + 1.04 \cdot 10^{-4} F (\ln r + 0.5)$$

$$\frac{\sigma_1}{r} = c_{11} + \frac{c_{12}}{r^2} + 1.04 \cdot 10^{-4} F (\ln r - 0.5)$$

$$M_{r1} = -B_1 \left(\frac{d\sigma_1}{dr} + \nu \frac{\sigma_1}{r} \right) = -1.92 \cdot 10^4 \left(c_{11} - \frac{c_{12}}{r^2} + 1.04 \cdot 10^{-4} F (\ln r + 0.5) + 0.3 \cdot c_{11} + 0.3 \frac{c_{12}}{r^2} + 3.12 \cdot 10^{-5} F (\ln r - 0.5) \right) = -1.92 \cdot 10^4 \left(1.3c_{11} - 0.4 \frac{c_{12}}{r^2} + 1.35 \cdot 10^{-4} F \ln r + 3.64 \cdot 10^{-5} F \right)$$

$$w_1 = \int \sigma_1 dr + C_{31} = 0.5 \cdot c_{11} r^2 + c_{12} \ln r + 1.08 \cdot 10^{-4} F \int r \ln r dr - 5.2 \cdot 10^{-5} F \int r dr + C_{31} = 0.5 c_{11} r^2 + c_{12} \ln r + 5.4 \cdot 10^{-5} F r^2 \ln r - 5.3 \cdot 10^{-5} r^2 + C_{31}$$

$$\sigma_2 = c_{21}r + \frac{c_{22}}{r} + 1.3 \cdot 10^{-4} F r (\ln r - 0.5)$$

$$\frac{d\sigma_2}{dr} = c_{21} - \frac{c_{22}}{r^2} + 1.3 \cdot 10^{-4} F (\ln r + 0.5), \quad \frac{\sigma_2}{r} = c_{21} + \frac{c_{22}}{r^2} + 1.3 \cdot 10^{-4} F (\ln r - 0.5)$$

$$M_{r2} = -B_2 \left(\frac{d\sigma_2}{dr} + \nu \frac{\sigma_2}{r} \right) = -1.54 \cdot 10^5 \left(c_{21} - \frac{c_{22}}{r^2} + 1.3 \cdot 10^{-4} F (\ln r + 0.5) + 0.3 c_{21} + 0.3 \frac{c_{22}}{r^2} + 3.9 \cdot 10^{-5} F (\ln r - 0.5) \right) = -1.54 \cdot 10^5 \left(1.3c_{21} - 0.4 \frac{c_{22}}{r^2} + 1.69 \cdot 10^{-4} F \ln r + 4.55 \cdot 10^{-5} F \right)$$

$$w_2 = \int \sigma_2 dr + C_{32} = 0.5 c_{21} r^2 + c_{22} \ln r + 1.3 \cdot 10^{-4} F \int r \ln r dr - 6.5 \cdot 10^{-5} F \int r dr + C_{32} = 0.5 c_{21} r^2 + c_{22} \ln r + 1.3 \cdot 10^{-4} F \cdot 0.5 \cdot r^2 (\ln r - 0.5) - 6.5 \cdot 10^{-5} F \cdot 0.5 \cdot r^2 + C_{32} = 0.5 c_{21} r^2 + c_{22} \ln r + 6.5 \cdot 10^{-5} F r^2 \ln r - 6.5 \cdot 10^{-5} r^2 + C_{32}$$

5. Становення $c_{11}, c_{12}, c_{21}, c_{22}$ а C_{31} а C_{32}

$$c_{11} \cdot 20 + c_{12} \frac{1}{20} + 1.04 \cdot 10^{-4} F \ln 20 - 0.5 = 0$$

$$0.5 \cdot c_{11} \cdot 20^2 + c_{12} \ln 20 + 5.4 \cdot 10^{-5} F \cdot 20^2 \ln 20 - 5.3 \cdot 10^{-5} 20^2 + C_{31} = 0$$

$$-1.92 \cdot 10^4 \left(1.3 \cdot c_{11} - 0.4 \frac{c_{12}}{40^2} + 1.35 \cdot 10^{-4} F \ln 40 + 3.64 \cdot 10^{-5} F \right) = 0$$

$$-1.54 \cdot 10^5 \left(1.3 c_{21} - 0.4 \frac{c_{22}}{40^2} + 1.69 \cdot 10^{-4} F \ln 40 + 4.55 \cdot 10^{-5} F \right) = 0$$

$$-1.54 \cdot 10^5 \left(1.3 c_{21} - 0.4 \frac{c_{22}}{60^2} + 1.69 \cdot 10^{-4} F \ln 60 + 4.55 \cdot 10^{-5} F \right) = 0$$

$$0.5 c_{21} \cdot 60^2 + c_{22} \ln 60 + 6.5 \cdot 10^{-5} F \cdot 60^2 \ln 60 - 6.5 \cdot 10^{-5} 60^2 + C_{32} = 0$$

$$| 0.5 c_{11} 40^2 + c_{12} \ln 40 + 5.4 \cdot 10^{-5} F \cdot 40^2 \ln 40 - 5.3 \cdot 10^{-5} 40^2 + C_{31} | + | 0.5 c_{21} 40^2 + c_{22} \ln 40 + 6.5 \cdot 10^{-5} F \cdot 40^2 \ln 40 - 6.5 \cdot 10^{-5} 40^2 + C_{32} | = 3$$

$$\begin{bmatrix} 20 & 5.0 \cdot 10^{-2} & 0 & 0 & 0 & 0 & 5.19 \cdot 10^{-3} \\ 2 \cdot 10^2 & 3 & 1 & 0 & 0 & 0 & 6.44 \cdot 10^{-2} \\ 13 & -4.38 \cdot 10^{-4} & 0 & 0 & 0 & 0 & 5.34 \cdot 10^{-4} \\ 0 & 0 & 0 & 1.3 & -4.38 \cdot 10^{-4} & 0 & 6.65 \cdot 10^{-4} \\ 0 & 0 & 0 & 1.3 & -1.94 \cdot 10^{-4} & 0 & 4.34 \cdot 10^{-4} \\ 0 & 0 & 0 & 1.8 \cdot 10^3 & 4.09 & 1 & 9.58 \cdot 10^{-1} \\ -8 \cdot 10^2 & -3.69 & -1 & 8 \cdot 10^2 & 3.69 & 1 & 6.49 \cdot 10^{-2} \end{bmatrix} \begin{bmatrix} C_{11} \\ C_{12} \\ C_{31} \\ C_{21} \\ C_{22} \\ C_{32} \\ F \end{bmatrix} = \begin{bmatrix} 0 \\ +2.12 \cdot 10^{-2} \\ 0 \\ 0 \\ 0 \\ 2.34 \cdot 10^{-1} \\ 3.62 \end{bmatrix}$$

C. Rechen

$$\Rightarrow \left. \begin{aligned} C_{11} &= -1.22 \cdot 10^{-2} \\ C_{12} &= 1.65 \\ C_{31} &= -4.5 \\ C_{21} &= -1.88 \cdot 10^{-2} \\ C_{22} &= -8.63 \\ C_{32} &= 29.49 \\ F &= 30.94 \end{aligned} \right\} \Rightarrow$$

$$M_{r1} = -1.92 \cdot 10^4 \left(1.3 \cdot (-1.22 \cdot 10^{-2}) - 0.4 \cdot 1.65 \cdot \frac{1}{r^2} + 1.35 \cdot 10^{-4} \cdot 30.94 \cdot \ln r + 3.64 \cdot 10^{-5} \cdot 30.94 \right) = -1.92 \cdot 10^4 \cdot$$

$$\cdot \left(-2.08 \cdot 10^{-2} - 1.16 \cdot \frac{1}{r^2} + 4.18 \cdot 10^{-3} \ln r \right)$$

$$M_{t1} = -1.92 \cdot 10^4 \left[(-1.22 \cdot 10^{-2}) + 1.65 \cdot \frac{1}{r^2} + 1.04 \cdot 10^{-4} \cdot 30.94 \cdot$$

$$\cdot (\ln r - 0.5) + 0.3 \cdot (-1.22 \cdot 10^{-2}) - 0.3 \cdot 1.65 \cdot \frac{1}{r^2} + 0.3 \cdot 1.04 \cdot 10^{-4} \cdot 30.94 \cdot (\ln r + 0.5) \right] = -1.92 \cdot 10^4 \left(-1.4 \cdot 10^{-2} + 1.76 \cdot \frac{1}{r^2} + 4.19 \cdot 10^{-3} \ln r \right)$$

$$M_{r2} = -1.54 \cdot 10^5 \left(1.3 \cdot (-1.88 \cdot 10^{-2}) - 0.4 \cdot (-8.63) \cdot \frac{1}{r^2} + 1.69 \cdot 10^{-4} \cdot 30.94 \ln r + 4.55 \cdot 10^{-5} \cdot 30.94 \right) = -1.54 \cdot 10^5 \left(-2.3 \cdot 10^{-2} + 6.04 \cdot \frac{1}{r^2} + 5.23 \cdot 10^{-3} \ln r \right)$$

$$M_{t2} = -1.54 \cdot 10^5 \left(-1.88 \cdot 10^{-2} + (-8.63) \cdot \frac{1}{r^2} + 1.3 \cdot 10^{-4} \cdot 30.94 \cdot (\ln r - 0.5) + 0.3 \cdot (-1.88 \cdot 10^{-2}) - 0.3 \cdot (-8.63) \cdot \frac{1}{r^2} + 0.3 \cdot 1.3 \cdot 10^{-4} \cdot 30.94 \cdot (\ln r + 0.5) \right) = -1.54 \cdot 10^5 \left(-2.58 \cdot 10^{-2} - 6.04 \cdot \frac{1}{r^2} + 5.23 \cdot 10^{-3} \ln r \right)$$

$$\sigma'_{r,el} = \pm \frac{6 M_{r1}}{h^2} = \dots, \quad \sigma'_{t,el} = \pm \frac{6 M_{t1}}{h^2}$$

$$\sigma'_{r,el} = \pm \frac{6 M_{r2}}{h^2} = \dots, \quad \sigma'_{t,el} = \pm \frac{6 M_{t2}}{h^2}$$

