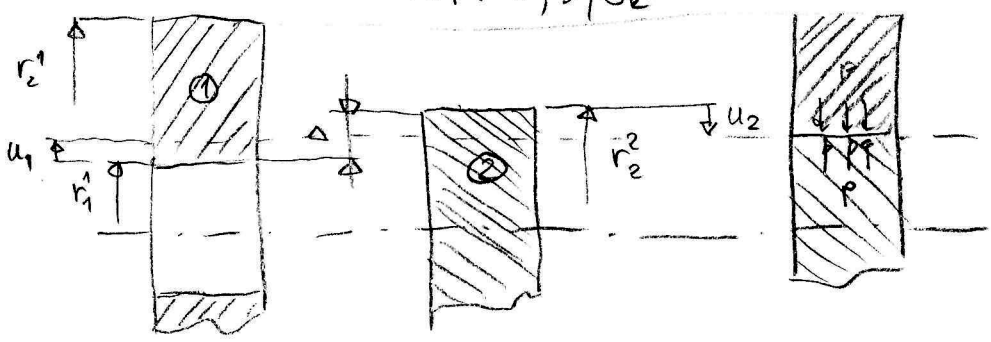
Př: Uteles podle obr stanovte bezpečnost vzhledem k MS pružnosti

mat: E, ν, σ_k



der. podm. ①:

$$\sigma_r^1|_{r=r_1^1} = -p$$

$$\sigma_r^1|_{r=r_2^1} = 0 \;,\; \sigma_t^1 = 0$$

der. podm. ②:

$$\sigma_r^2|_{r=r_2^2} = -p \;,\; \sigma_t^2 = 0$$

def. podmínka: $|u_1| + |u_2| = \Delta \Rightarrow p$

těleso ①

$$\sigma_r^1 = A_1 - \frac{B_1}{r^2} \Rightarrow A_1 - \frac{B_1}{(r_1^1)^2} = -p \;,\; A_1 - \frac{B_1}{(r_2^1)^2} = 0 \Rightarrow$$

$$\Rightarrow B_1\left(-\frac{1}{(r_1^1)^2} + \frac{1}{(r_2^1)^2}\right) = -p \Rightarrow B_1 = -\frac{p}{-\frac{1}{(r_1^1)^2} + \frac{1}{(r_2^1)^2}} = -\frac{p(r_1^1)^2(r_2^1)^2}{(r_1^1)^2 - (r_2^1)^2}$$

$$\Rightarrow A_1 = \frac{B_1}{(r_2^1)^2} = -\frac{p(r_1^1)^2}{(r_1^1)^2 - (r_2^1)^2}$$

$$\sigma_r^1 = -\frac{p(r_1^1)^2}{(r_1^1)^2 - (r_2^1)^2} + \frac{p(r_1^1)^2(r_2^1)^2}{(r_1^1)^2 - (r_2^1)^2}\frac{1}{r^2} = -\frac{p(r_1^1)^2}{(r_1^1)^2 - (r_2^1)^2}\left(1 - \frac{(r_2^1)^2}{r^2}\right)$$

$$\sigma_t^1 = -\frac{p(r_1^1)^2}{(r_1^1)^2 - (r_2^1)^2}\left(1 + \frac{(r_2^1)^2}{r^2}\right)$$

$$u_1 = \frac{(1+\nu)(1-2\nu)}{E} \cdot \frac{-p(r_1^1)^2}{(r_1^1)^2 - (r_2^1)^2} \cdot r + \frac{1+\nu}{E} \cdot \frac{-p(r_1^1)^2(r_2^1)^2}{(r_1^1)^2 - (r_2^1)^2} \cdot \frac{1}{r}$$

$$= -\frac{p(r_1^1)^2}{(r_1^1)^2 - (r_2^1)^2}\left[\frac{(1+\nu)(1-2\nu)}{E} \cdot r + \frac{1+\nu}{E}\frac{(r_2^1)^2}{r}\right]$$

těleso ②

$$\sigma_r^2 = A_2 \Rightarrow A_2 = -p$$

$$\sigma_r^2 = A_2 = -p$$

$$\sigma_t^2 = A_2 = -p$$

$$u_2 = \frac{(1+\nu)(1-2\nu)}{E} \cdot (-p) \cdot r$$

těleso ① + ②

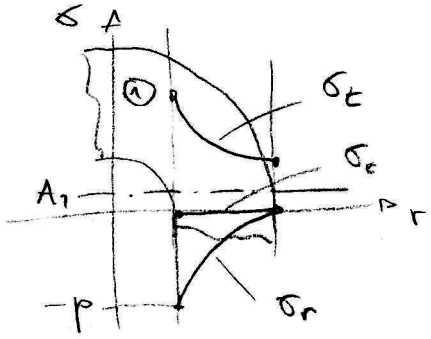
$$|u_1|_{r=r_1^1} + |u_2|_{r=r_2^2} = \Delta$$

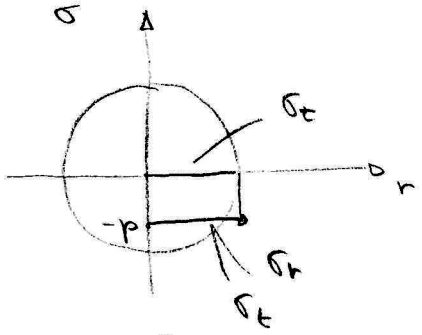
$$-\frac{p(r_1^1)^2}{(r_1^1)^2 - (r_2^1)^2}\left[\frac{(1+\nu)(1-2\nu)}{E} r_1^1 + \frac{1+\nu}{E}\frac{(r_2^1)^2}{r_1^1}\right] + \frac{(1+\nu)(1-2\nu)}{E} \cdot p \cdot r_2^2 = \Delta$$

$$p\left\{(r_1')^2\left[(1+\nu)(1-2\nu)(r_1')^2+(1+\nu)(r_2')^2\right]+(1+\nu)(1-2\nu)\,r_2^2 r_1'\left[(r_1')^2-(r_2')^2\right]\right\}$$

$$= \Delta r_1'\left((r_1')^2-(r_2')^2\right)$$

$$p=\frac{\Delta r_1'\left[(r_1')^2-(r_2')^2\right]}{(1+\nu)(1-2\nu)\left[(r_1')^4+r_1' r_2^2\left[(r_1')^2-(r_2')^2\right]\right]+(1+\nu)(r_1')^2(r_2')^2}$$





$$\sigma_{red}^1 = \left|\sigma_t^1-\sigma_r^1\right|\Big|_{r=r_1} = -2\,\frac{p\cancel{(r_1')^2}}{(r_1')^2-(r_2')^2}\,\frac{(r_2')^2}{\cancel{(r_1')^2}} = -2\,\frac{p\,(r_2')^2}{(r_1')^2-(r_2')^2}$$

$$\sigma_{red}^2 = \left|\sigma_t^2-\sigma_z^2\right| = \left|\sigma_r^2-\sigma_z^2\right| = p$$

$$k=\min\left\{\frac{\sigma_k}{\sigma_{red}^1}\,;\,\frac{\sigma_k}{\sigma_{red}^2}\right\}$$