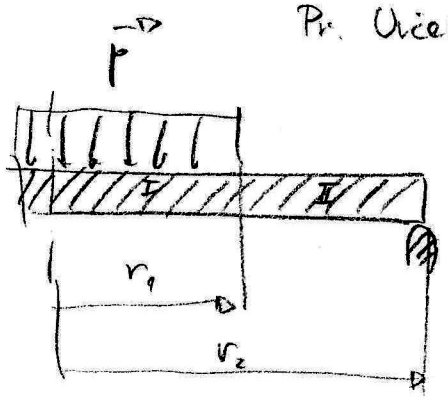
Př. Určete bezpečnost k MS pružnosti a w_{max}



mat.: $\nu = 0.3$
$E = 2.1 \cdot 10^5$ MPa

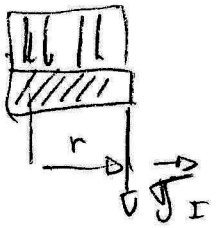
geom.: $r_1 = 10$ mm
$r_2 = 20$ mm
$h = 1$ mm

zatížení:

$p r_1^2 \pi = 1 \Rightarrow p = \frac{1}{\pi \cdot 10^2} = 3.18 \cdot 10^{-3}$

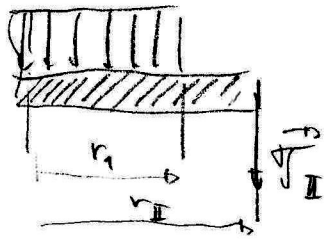
tuhost desky: $B = \frac{E h^3}{12(1-\nu^2)} = \frac{2.1 \cdot 10^5 \cdot 1^3}{12(1-0.3^2)} = 1.92 \cdot 10^4$ N/mm

1. Uložení elementu:



$r_I < r_1$

$2\pi r_I \sigma_I = -\pi r_I^2 p \Rightarrow \sigma_I = -\frac{1}{2} r_I p = -0.5 \cdot 3.18 \cdot 10^{-3} r_I = -1.59 \cdot 10^{-3} r_I$



$r_1 < r_{II} < r_2$

$2\pi r_{II} \sigma_{II} = -\pi r_1^2 p \Rightarrow \sigma_{II} = -\frac{r_1^2 p}{2 r_{II}} = -0.5 \cdot 10^2 \cdot 3.18 \cdot 10^{-3} \frac{1}{r_{II}}$

$= -1.59 \cdot 10^{-1} \frac{1}{r_{II}}$

2. Partikulární řešení

$$\vartheta_p^I = -\frac{1}{r_I B} \int r_I \int \sigma_I \, dr_I dr_I = -\frac{1}{r_I B} \int r_I \int -1.59 \cdot 10^{-3} r_I \, dr_I dr_I = 1.59 \cdot 10^{-3} \frac{1}{1.92 \cdot 10^4} \cdot \frac{1}{r_I} \cdot$$

$$\cdot \int r_I \, 0.5 r_I^2 dr_I = 8.28 \cdot 10^{-8} \frac{1}{r_I} \cdot 0.5 \cdot 0.25 \cdot r_I^4 = 1.04 \cdot 10^{-8} r_I^3$$

$$\vartheta_p^{II} = -\frac{1}{r_{II} B} \int r_{II} \int \sigma_{II} \, dr_{II} dr_{II} = -\frac{1}{r_{II} \cdot 1.92 \cdot 10^4} \int r_{II} \int -1.59 \cdot 10^{-1} \frac{1}{r_{II}} dr_{II} dr_{II} = 8.28 \cdot 10^{-6} \cdot \frac{1}{r_{II}}$$

$$\cdot \int r_{II} \ln r_{II} \, dr_{II} = 8.28 \cdot 10^{-6} \frac{1}{r_{II}} \cdot 0.5 \cdot r_{II}^2 (\ln r_{II} - 0.5) = 4.14 \cdot 10^{-6} r_{II} (\ln r_{II} - 0.5)$$

3. Natočení, momenty, průhyb

$$\vartheta_I = C_1^I r_I + \vartheta_p^I = C_1^I r_I + 1.04 \cdot 10^{-8} r_I^3$$

$$\vartheta_{II} = C_1^{II} r_{II} + C_2^{II} \frac{1}{r_{II}} + \vartheta_p^{II} = C_1^{II} r_{II} + C_2^{II} \frac{1}{r_{II}} + 4.14 \cdot 10^{-6} r_{II} (\ln r_{II} - 0.5)$$

$$\frac{d\vartheta_I}{dr_I} = C_1^I + 1.04 \cdot 10^{-8} \cdot 3 r_I^2 = C_1^I + 3.12 \cdot 10^{-8} r_I^2$$

$$\frac{d\vartheta_{II}}{dr_{II}} = C_1^{II} - C_2^{II} \frac{1}{r_{II}^2} + 4.14 \cdot 10^{-6} (\ln r_{II} - 0.5) + 4.14 \cdot 10^{-6} r_{II} \cdot \frac{1}{r_{II}} = C_1^{II} - C_2^{II} \frac{1}{r_{II}^2} + 4.14 \cdot 10^{-6} \ln r_{II} +$$

$+2.09\cdot10^{-6}$

$$\frac{\vartheta_I}{r_I} = C_1^I + 1.04\cdot10^{-8} r_I^2$$

$$\frac{\vartheta_{II}}{r_{II}} = C_1^{II} + C_2^{II}\frac{1}{r_{II}^2} + 4.14\,10^{-6}(\ln r_{II} - 0.5) = C_1^{II} + C_2^{II}\frac{1}{r_{II}^2} + 4.14\cdot10^{-6}\ln r_{II} - 2.09\cdot10^{-6}$$

$$M_r^I = -B\left(\frac{d\vartheta_I}{dr_I} + v\frac{\vartheta_I}{r_I}\right) = -1.92\cdot10^4\left(C_1^I + 3.12\cdot10^{-8} r_I^2 + 0.3\left(C_1^I + 1.04\cdot10^{-8} r_I^2\right)\right)$$

$$= -1.92\cdot10^4\left(1.3\,C_1^I + 3.43\cdot10^{-8} r_I^2\right)$$

$$M_r^{II} = -B\left(\frac{d\vartheta_{II}}{dr_{II}} + v\frac{\vartheta_{II}}{r_{II}}\right) = -1.92\cdot10^4\left(C_1^{II} - C_2^{II}\cdot\frac{1}{r_{II}^2} + 4.14\cdot10^{-6}\ln r_{II} + 2.09\cdot10^{-6} + 0.3\left(C_1^{II} + C_2^{II}\cdot\frac{1}{r_{II}^2} + 4.14\cdot10^{-6}\ln r_{II} - 2.09\cdot10^{-6}\right)\right) = -1.92\cdot10^4\left(1.3C_1^{II} - 0.7\frac{C_2^{II}}{r_{II}^2} + 5.38\cdot10^{-6}\ln r_{II} + 1.45\cdot10^{-6}\right)$$

$$w_I = \int \vartheta_I\,dr_I + C_3^I = \int\left(C_1^I r_I + 1.04\cdot10^{-8} r_I^3\right)dr_I + C_3^I = C_1^I\,0.5\,r_I^2 + 1.04\cdot10^{-8}\cdot0.25\,r_I^4 + C_3^I = 0.5\,r_I^2 C_1^I + 2.6\cdot10^{-9} r_I^4 + C_3^I$$

$$w_{II} = \int \vartheta_{II}\,dr_{II} + C_3^{II} = \int\left(C_1^{II} r_{II} + C_2^{II}\frac{1}{r_{II}} + 4.14\cdot10^{-6} r_{II}\ln r_{II} - 2.09\cdot10^{-6} r_{II}\right)dr_{II} + C_3^{II} =$$

$$= 0.5\,C_1^{II} r_{II}^2 + \ln r_{II}\,C_2^{II} + 4.14\,10^{-6}\,0.5\,r_{II}^2(\ln r_{II} - 0.5) - 2.09\cdot10^{-6}\cdot0.5\,r_{II}^2 + C_3^{II} =$$

$$= 0.5\cdot r_{II}^2\cdot C_1^{II} + \ln r_{II}\cdot C_2^{II} + 2.09\cdot10^{-6} r_{II}^2\ln r_{II} - 2.09\cdot10^{-6} r_{II}^2 + C_3^{II}$$

4. Okrajové podmínky

- $\vartheta_I|_{r=r_1} = \vartheta_{II}|_{r=r_{II}}$
- $M_r^I|_{r_I=r_1} = M_r^{II}|_{r_{II}=r_1}$
- $w_{II}|_{r_{II}=r_2} = 0$
- $w_I|_{r=r_I} = w_{II}|_{r=r_{II}}$
- $M_r^{II}|_{r_{II}=r_2} = 0$

5. Stanovení $C_1^I, C_2^{II}, C_3^I, C_1^{II}, C_3^{II}$:

$$C_1^I\cdot10 + 1.04\cdot10^{-8}\cdot10^3 = C_1^{II}\,10 + C_2^{II}\cdot\frac{1}{10} + 4.14\cdot10^{-6}\cdot10(\ln10 - 0.5)$$

$$-1.92\cdot10^4\left(1.3C_1^I + 3.43\,10^{-8}\cdot10^2\right) = -1.92\cdot10^4\left(1.3\cdot C_1^{II} - 0.7\cdot\frac{C_2^{II}}{10^2} + 5.38\cdot10^{-6}\ln10 + 1.45\cdot10^{-6}\right)$$

$$-1.92\cdot10^4\left(1.3\cdot C_1^{II} - 0.7\cdot\frac{C_2^{II}}{20^2} + 5.38\cdot10^{-6}\ln20 + 1.45\cdot10^{-6}\right) = 0$$

$$10\,C_1^I - 10\,C_1^{II} - 0.1\cdot C_2^{II} = 4.14\cdot10^{-6}\cdot10(\ln10 - 0.5) - 1.04\cdot10^{-8}\cdot10^3 = 6.42\cdot10^{-5}$$

$$1.3\,C_1^I - 1.3\,C_1^{II} + 4\cdot10^{-3}\,C_2^{II} = 5.38\cdot10^{-6}\ln10 + 1.45\cdot10^{-6} - 3.43\cdot10^{-8}\cdot10^2 = 1.04\cdot10^{-5}$$

$$1.3\,C_1^{II} - 1.45\cdot10^{-3}\,C_2^{II} = -5.38\cdot10^{-6}\ln20 - 1.45\cdot10^{-6} = -1.76\,10^{-5}$$

$$C_1^{II} = \frac{-1.76\cdot10^{-5} + 1.75\cdot10^{-3} C_2^{I}}{1.3}$$

$$10\cdot C_1^{I} - \frac{10}{1.3}\left(-1.76\cdot10^{-5} + 1.75\cdot10^{-3} C_2^{II}\right) - 0.1\cdot C_2^{II} = 6.42\cdot10^{-5}$$

$$1.3\, C_1^{I} - \left(-1.76\cdot10^{-5} + 1.75\cdot10^{-3} C_2^{II}\right) + 4\cdot10^{-3} C_2^{II} = 1.04\cdot10^{-5}$$

$$10\, C_1^{I} - 1.13\cdot10^{-1} C_2^{II} = 6.42\cdot10^{-5} - \frac{10}{1.3}\,1.76\cdot10^{-5} = -7.12\cdot10^{-5}$$

$$1.3 C_1^{I} + 5.25\cdot10^{-3} C_2^{II} = 1.04\cdot10^{-5} - 1.76\cdot10^{-5} = -7.2\cdot10^{-6}$$

$$C_2^{II} = \frac{-7.2\cdot10^{-6} - 1.3 C_1^{I}}{5.25\cdot10^{-3}}$$

$$10\cdot C_1^{I} - 1.13\cdot10^{-1}\cdot\frac{-7.2\cdot10^{-6} - 1.3 C_1^{I}}{5.25\cdot10^{-3}} = -7.12\cdot10^{-5}$$

$$\left(10 + 1.13\cdot10^{-1}\,\frac{1.3}{5.25\cdot10^{-3}}\right) C_1^{I} = -7.12\cdot10^{-5} - 1.13\cdot10^{-1}\,\frac{7.2\cdot10^{-6}}{5.25\cdot10^{-3}}$$

$$38 C_1^{I} = -2.26\cdot10^{-4}$$

$$C_1^{I} = -5.95\cdot10^{-6}$$

$$C_2^{II} = \frac{-7.2\cdot10^{-6} - 1.3\cdot(-595\cdot10^{-6})}{5.25\cdot10^{-3}} = 1.02\cdot10^{-4}$$

$$C_1^{II} = \frac{-1.76\cdot10^{-5} + 1.75\cdot10^{-3}\cdot1.02\cdot10^{-4}}{1.3} = -1.34\cdot10^{-5}$$

$$0.5\cdot10^{2} C_1^{I} + 2.6\cdot10^{-9}\cdot10^{4} + C_3^{I} = 0.5\cdot10^{2}\cdot C_1^{II} + \ln 10\, C_2^{II} + 2.07\cdot10^{-6}\,10^{2}\ln 10 - 2.07\cdot10^{-6}\,10^{2} + C_3^{II}$$

$$0 = 0.5\cdot20^{2} C_1^{II} + \ln 20\, C_2^{II} + 2.07\cdot10^{-6}\cdot20^{2}\ln 20 - 2.07\cdot10^{-6}\,20^{2} + C_3^{II}$$

$$C_3^{II} = -0.5\cdot20^{2}\cdot(-1.34\cdot10^{-5}) - \ln 20\cdot 1.02\cdot10^{-4} - 2.07\cdot10^{-6}\,20^{2}\ln 20 + 2.07\cdot10^{-6}\,20^{2}$$

$$= 4.22\cdot10^{-4}$$

$$C_3^{I} = 0.5\cdot10^{2}\cdot(-1.34\cdot10^{-5}) + \ln 10\cdot 1.02\cdot10^{-4} + 2.07\cdot10^{-6}\cdot10^{2}\ln 10 - 2.07\cdot10^{-6}\cdot10^{2} + 4.22\cdot10^{-4} - 0.5\cdot10^{2}\cdot(-5.95\cdot10^{-6}) - 2.6\cdot10^{-9}\cdot10^{4} = 8.28\cdot10^{-4}$$

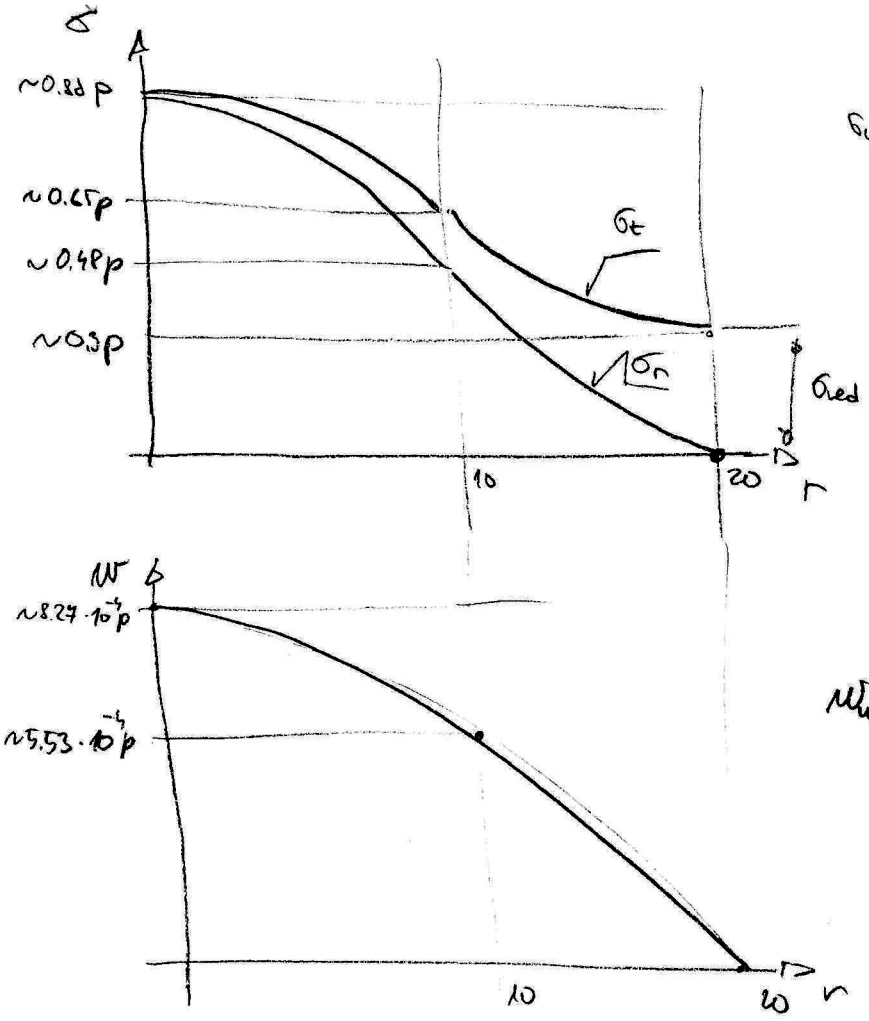
6. Resen

$$\sigma_{r,ek}^{I} = \frac{6 M_r^{I}}{h^2} = -1.92\cdot10^{4}\cdot6\left(1.3\cdot(-5.95\cdot10^{-6}) + 3.43\cdot10^{-8}\cdot r_I^{2}\right) =$$

$$= -1.15\cdot10^{5}\left(-4.74\cdot10^{-6} + 3.43\,10^{-8} r_I^{2}\right)$$

$$\sigma_{r,ek}^{II} = \frac{6 M_r^{II}}{h^2} = -1.92\cdot10^{4}\cdot6\left(1.3\cdot(-1.34\cdot10^{-5}) - 0.4\cdot1.02\cdot10^{-4}\cdot\frac{1}{r_2} + 5.38\cdot10^{-6}\ln r_{II} + 1.45\cdot10^{-6}\right)$$

$$= -1.15\cdot10^{5}\left(-1.74\cdot10^{-5} - 4.14\cdot10^{-5}\,\frac{1}{r_{II}^{2}} + 5.38\cdot10^{-6}\ln r_{II} + 1.45\cdot10^{-6}\right)$$



$$\sigma_{red} = |\sigma_{r,ea} - \sigma_{t,ea}|\Big|_{r_{II}=20} \cong 0.3p \text{ MPa}$$

$$k = \frac{\sigma_k}{\sigma_{red}}$$

$$w_{max} = w^I\Big|_{r=0} = 8.28 \cdot 10^{-4} p \text{ mm}$$

$$M_t^I = -B\left(\frac{\nu_I}{r_I} + v\frac{d\nu_I}{dr_I}\right) = -1.92\cdot 10^4\left(-5.95\cdot 10^{-6} + 1.04\cdot 10^{-8}\cdot r_I^2 + 0.3(-5.95\cdot 10^{-6} + 3.12\cdot 10^{-8} r_I^2)\right) = -1.92\cdot 10^4\left(-7.74e^{-6} + 1.98\cdot 10^{-8} r_I^2\right)$$

$$M_t^{II} = -B\left(\frac{\nu_{II}}{r_{II}} + v\frac{d\nu_{II}}{dr_{II}}\right) = -1.92\cdot 10^4\left(-1.34\cdot 10^{-5} + 1.02\cdot 10^{-4}\frac{1}{r_{II}^2} + 4.14\cdot 10^{-6}\ln r_{II} - 2.07\cdot 10^{-6} + 0.3\left(-1.34\cdot 10^{-5} - 1.02\cdot 10^{-4}\frac{1}{r_{II}^2} + 4.14\cdot 10^{-6}\ln r_{II} + 2.07\cdot 10^{-6}\right)\right)$$

$$= -1.92\cdot 10^4\left(-1.89\cdot 10^{-5} + 4.14\cdot 10^{-5}\frac{1}{r_{II}^2} + 5.38\cdot 10^{-6}\ln r_{II}\right)$$

$$\sigma_{t,ea}^I = \frac{6M_t^I}{h^2} = 6\cdot(-1.92\cdot 10^4)\left(-7.74\cdot 10^{-6} + 1.98\cdot 10^{-8} r_I^2\right) = -1.15\cdot 10^5(\ldots$$

$$\sigma_{t,ea}^{II} = \frac{6M_t^{II}}{h^2} = 6\cdot(-1.92\cdot 10^4)\cdot\left(-1.89\cdot 10^{-5} + 4.14\cdot 10^{-5}\frac{1}{r_{II}^2} + 5.38\cdot 10^{-6}\ln r_{II}\right) = -1.15\cdot 10^5\cdot(\ldots$$

$$w^I = 0.5\cdot r_I^2\cdot(-5.95\cdot 10^{-6}) + 2.6\cdot 10^{-9}\cdot r_I^4 + (-1.29\cdot 10^2) + 8.28\cdot 10^{-4} = -2.98\cdot 10^{-6} r_I^2 + 2.6\cdot 10^{-9}\cdot r_I^4 +$$

$$w^{II} = 0.5\cdot r_{II}^2(-1.34\cdot 10^{-5}) + \ln r_{II}\cdot 1.02\cdot 10^{-4} + 2.07\cdot 10^{-6} r_{II}^2\ln r_{II} - 2.07\cdot 10^{-6} r_{II}^2 + (-1.29\cdot 10^2) = -8.47\cdot 10^{-6} r_{II}^2 + 1.02\cdot 10^{-4}\ln r_I + 2.07\cdot 10^{-6} r_{II}^2\ln r_{II} + 7.22\cdot 10^{-4}$$