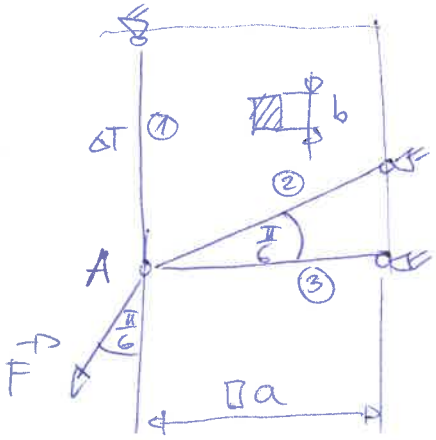
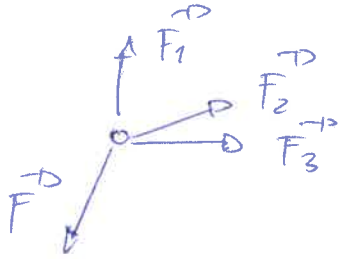


Dáno: $a, b, F, \Delta T, E, \sigma_k, \alpha_T$

Určete: k, $\vec{u}_A$

1. Úplné uvolnění



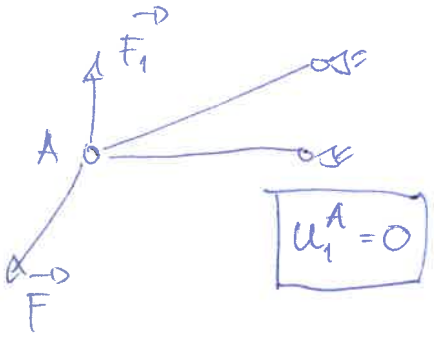
2. Statický určení

$NP = \{F_1, F_2, F_3\}$

$\left.\begin{matrix} \mu = 3 \\ v = 2 \end{matrix}\right\} \; s = \mu - v = 1$

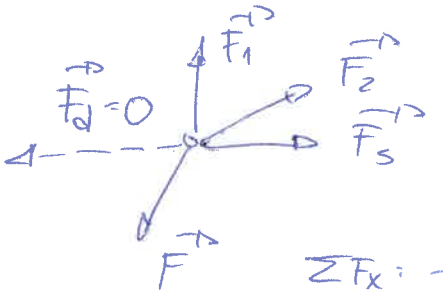
Úloha 1x stat. neurč.

3. Částečné uvolnění



$u_1^A = 0$

4. VVÚ



Zavedení doplňkové síly z důvodu určení $\vec{u}_A$.

$\Sigma F_x: \; -F_d + F_2 \cdot \frac{\sqrt{3}}{2} + F_3 - F \cdot \frac{1}{2} = 0$

$\Sigma F_y: \; F_1 + F_2 \cdot \frac{1}{2} - F \cdot \frac{\sqrt{3}}{2} = 0$

$F_2 = \sqrt{3}F - 2F_1$

$-F_d + \frac{\sqrt{3}}{2}(\sqrt{3}F - 2F_1) + F_3 - \frac{1}{2}F = 0$

$F_2 = \sqrt{3}F - 2F_1$

$F_3 = F_d - F + \sqrt{3}F_1$

$N_1 = F_1 \Rightarrow \frac{\partial N_1}{\partial F_1} = 1$

$N_2 = F_2 = \sqrt{3}F - 2F_1 \Rightarrow \frac{\partial N_2}{\partial F_1} = -2$

$N_3 = F_3 = F_d - F + \sqrt{3}F_1 \Rightarrow \frac{\partial N_3}{\partial F_1} = \sqrt{3}$

5. Odstranění stat. neurč.

$$u_1^A = 0$$

$$\sum_{i=1}^{3} \frac{N_i l_i}{ES} \frac{\partial N_i}{\partial F_1} + \Delta T \alpha_T \cdot l_1 = 0$$

$l_1 = a, \; l_2 = \frac{a}{\frac{\sqrt{3}}{2}} = \frac{2\sqrt{3}a}{3}, \; l_3 = a$

$$F_1 \cdot \frac{1}{ES} \cdot 1 + (\sqrt{3}F - 2F_1) \cdot \frac{2\sqrt{3}}{3} \frac{1}{ES} \cdot (-2) + (F_d - F + \sqrt{3}F_1) \cdot \frac{1}{ES} \cdot \sqrt{3} + \Delta T \alpha_T \cdot \frac{1}{ES} \cdot E \cdot S = 0$$

$$F_1\left(1 + \frac{8\sqrt{3}}{3} + 3\right) + F\left(-4 - \sqrt{3}\right) + \sqrt{3}F_d = -\Delta T \alpha_T E \cdot S$$

$$F_1 = \frac{\sqrt{3}F_d + (4+\sqrt{3})F - \Delta T \alpha_T E \cdot S}{\frac{12 + 8\sqrt{3}}{3}}$$

$$F_1 = \frac{3(\sqrt{3}F_d + (4+\sqrt{3})F - \Delta T \alpha_T E \cdot S)}{12 + 8\sqrt{3}}$$

Řešení

$$N_1 = F_1 = \frac{3(\sqrt{3}F_d + (4+\sqrt{3})F - \Delta T \alpha_t E \cdot S)}{12+8\sqrt{3}} = \frac{3\sqrt{3}F_d + 3(4+\sqrt{3})F - 3\Delta T\alpha_t E\cdot S}{12+8\sqrt{3}}$$

$$N_2 = F_2 = \sqrt{3}F - \frac{3(\sqrt{3}F_d + (4+\sqrt{3})F - \Delta T\alpha_t E S)}{6\cdot(12+8\cdot\sqrt{3})} = \frac{\sqrt{3}(6+4\sqrt{3})F - 3\sqrt{3}F_d - 3(4+\sqrt{3})F}{6+4\sqrt{3}}$$

$$+ \frac{3\Delta T\alpha_t E S}{6+4\sqrt{3}} = \frac{-3\sqrt{3}F_d + 3\sqrt{3}F + 3\Delta T\alpha_t E S}{6+4\sqrt{3}}$$

$$N_3 = F_3 = F_d - F + \sqrt{3}\,\frac{3(\sqrt{3}F_d + (4+\sqrt{3})F - \Delta T\alpha_t E\cdot S)}{12+8\sqrt{3}} = \frac{(12+8\sqrt{3})F_d - (12+8\sqrt{3})F}{12+8\sqrt{3}}$$

$$+ \frac{3\sqrt{3}(\sqrt{3}F_d + (4+\sqrt{3})F - \Delta T\alpha_t E\cdot S)}{12+8\sqrt{3}} = \frac{(21+8\sqrt{3})F_d + (-3+4\sqrt{3})F - 3\sqrt{3}\Delta T\alpha_t E\cdot S}{12+8\sqrt{3}}$$

(i) $k = ?$, $F_d = 0$

$$N_1 = \frac{(12+3\sqrt{3})F - 3\Delta T\alpha_t E\cdot S}{12+8\cdot\sqrt{3}}$$

$$N_2 = \frac{(\sqrt{3}F + 6\Delta T\alpha_t E\cdot S}{12+8\sqrt{3}}$$

$$N_3 = \frac{(-3+4\sqrt{3})F - 3\sqrt{3}\Delta T\alpha_t E\cdot S}{12+8\sqrt{3}}$$

$N_{max} = max\{N_1, N_2, N_3\}$

záleží na teplotním zatížení, ale dá se odhadnout $N_{max} = N_1$

$$\sigma_{max} = \frac{N_1}{S} = \frac{(12+3\sqrt{3})F - 3\Delta T\alpha_t E\cdot S}{(12+8\sqrt{3})\cdot b^2}$$

$$k = \frac{\sigma_k}{\sigma_{max}} > 1$$

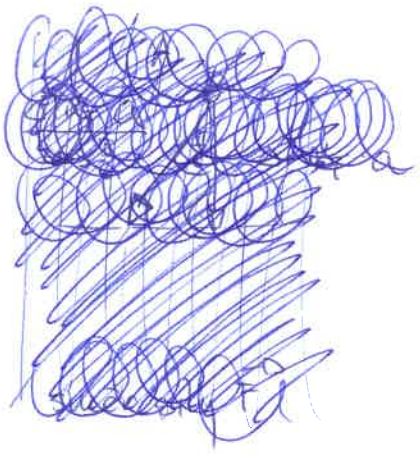
(ii) $\vec{u}_A = ?$

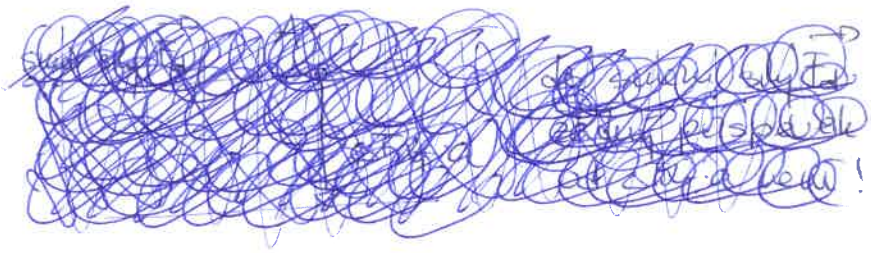
$$\frac{\partial N_1}{\partial F} = \frac{12+3\sqrt{3}}{12+8\sqrt{3}} \qquad \frac{\partial N_1}{\partial F_d} = \frac{3\sqrt{3}}{12+8\sqrt{3}}$$

$$\frac{\partial N_2}{\partial F} = \frac{6\sqrt{3}}{12+8\sqrt{3}} = \frac{3\sqrt{3}}{6+4\sqrt{3}} \qquad \frac{\partial N_2}{\partial F_d} = \frac{-3\sqrt{3}}{6+4\sqrt{3}}$$

$$\frac{\partial N_3}{\partial F} = \frac{-3+4\sqrt{3}}{12+8\sqrt{3}} \qquad \frac{\partial N_3}{\partial F_d} = \frac{21+8\sqrt{3}}{12+8\sqrt{3}}$$

Trispějeme prodloužení prutu vlivem teploty ΔT do směru síly $\vec{F}$ a $\vec{F_d}$:





Dále platí $F_d = 0$:

$$u_A^F = \sum_{i=1}^{3} \frac{N_i l_i}{ES} \cdot \frac{\partial N_i}{\partial F} = \frac{(12+3\sqrt{3})F - 3\sqrt{3}\alpha T E \cdot S}{(12+8\sqrt{3})E \cdot S} \cdot a \cdot \frac{12+3\sqrt{3}}{12+8\sqrt{3}} +$$

$$+ \frac{6\sqrt{3}F + 6\sqrt{3}\alpha T E \cdot S}{(12+8\sqrt{3})ES} \cdot \frac{\sqrt{3}\cdot 2a}{3} \cdot \frac{3\sqrt{3}}{6+4\sqrt{3}} + \frac{(-3+4\sqrt{3})F - 3\sqrt{3}\alpha T E \cdot S}{(12+8\sqrt{3})ES} \cdot a \cdot$$

$$\cdot \frac{-3+4\sqrt{3}}{12+8\sqrt{3}} + \ldots = \ldots$$

$$u_A^{F_d} = \sum_{i=1}^{3} \frac{N_i l_i}{ES} \frac{\partial N_i}{\partial F_d} + 0 = \frac{(12+3\sqrt{3})F - 3\sqrt{3}\alpha T E \cdot S}{(12+8\sqrt{3})ES} \cdot a \cdot \frac{3\sqrt{3}}{12+8\sqrt{3}} +$$

$$+ \frac{6\sqrt{3}F + 6\sqrt{3}\alpha T E \cdot S}{(12+8\sqrt{3})ES} \cdot \frac{\sqrt{3}\cdot 2a}{3} \cdot \frac{-3\sqrt{3}}{6+4\sqrt{3}} + \frac{(-3+4\sqrt{3})\cdot F - 3\sqrt{3}\alpha T E \cdot S}{(12+8\sqrt{3})ES} \cdot a \cdot$$

$$\cdot \frac{21+8\sqrt{3}}{12+8\sqrt{3}} - \ldots$$