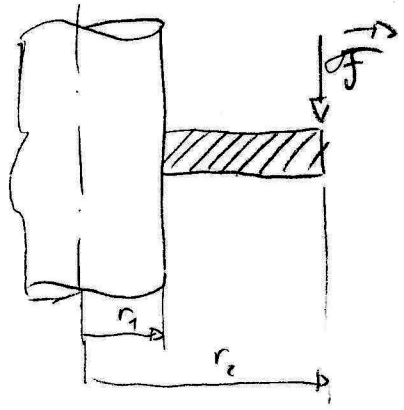
Př: Určete bezpečnost k MS pružnosti u desky podle obr. a max. průhyb



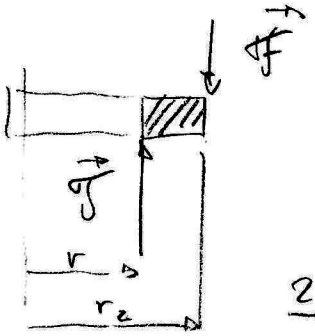
mat.: $\upsilon = 0.3$
$E = 2.1 \cdot 10^5 MPa$

geom.: $r_1 = 10mm$
$r_2 = 20mm$
$h = 1mm$

zatížení: $f_f = \frac{1}{2\pi r_2} = \frac{1}{2\pi \cdot 20} = 4.96 \cdot 10^{-3} \frac{N}{mm}$

tuhost desky: $B = \frac{E \cdot h^3}{12(1-\upsilon^2)} = \frac{2.1 \cdot 10^5 \cdot 1^3}{12(1-0.3^2)} = 1.92 \cdot 10^4 N \cdot mm$

1. Uložení elementu



$2\pi r T = 2\pi r_2 f = 1$

$T = \frac{1}{2\pi r} = 1.59 \cdot 10^{-1} \cdot \frac{1}{r}$

2. Partikulární řešení

$\vartheta_p = -\frac{1}{rB} \int r \int T \, dr \, dr = -\frac{1}{1.92 \cdot 10^4 r} \int r \int \left(1.59 \cdot 10^{-1} \frac{1}{r}\right) dr \, dr =$

$= -8.28 \cdot 10^{-6} \frac{1}{r} \int r \ln r \, dr = -8.28 \cdot 10^{-6} \frac{1}{r} \left[0.5 r^2 (\ln r - 0.5)\right] = -4.14 \cdot 10^{-6} r (\ln r - 0.5)$

3. Natočení, momenty a průhyb

$\vartheta = C_1 r + \frac{C_2}{r} - 4.14 \cdot 10^{-6} r (\ln r - 0.5)$

$\frac{d\vartheta}{dr} = C_1 + \left(-\frac{C_2}{r^2}\right) - 4.14 \cdot 10^{-6} (\ln r - 0.5) - 4.14 \cdot 10^{-6} r \left(\frac{1}{r}\right) = C_1 - \frac{C_2}{r^2} - 4.14 \cdot 10^{-6} (\ln r + 0.5)$

$\frac{\vartheta}{r} = C_1 + \frac{C_2}{r^2} - 4.14 \cdot 10^{-6} (\ln r - 0.5)$

$M_r = -B \left(\frac{d\vartheta}{dr} + \upsilon \frac{\vartheta}{r}\right) = -1.92 \cdot 10^4 \left(C_1 - \frac{C_2}{r^2} - 4.14 \cdot 10^{-6} (\ln r + 0.5) + 0.3 \left[C_1 + \frac{C_2}{r^2} - \right.\right.$

$\left.\left. - 4.14 \cdot 10^{-6} (\ln r - 0.5)\right]\right) = -1.92 \cdot 10^4 \left(1.3 C_1 - 0.7 \cdot \frac{C_2}{r^2} - 5.38 \cdot 10^{-6} \ln r - 1.45 \cdot 10^{-6}\right)$

$w = \int \vartheta \, dr + C_3 = C_1 r^2 \cdot 0.5 + C_2 \ln r - 4.14 \cdot 10^{-6} \left(0.5 r^2 (\ln r - 0.5) - 0.25 r^2\right) + C_3$

$= 0.5 r^2 \cdot C_1 + C_2 \ln r - 2.07 \cdot 10^{-6} r^2 \ln r + 2.04 \cdot 10^{-6} r^2 + C_3$

4. Okr. podmínky

$M_r|_{r=r_2} = 0, \quad \vartheta|_{r=r_1} = 0, \quad w|_{r=r_1} = 0$

5. Stanovení C_1, C_2, C_3

$$0 = 1.3 \cdot C_1 - 0.7 \cdot \frac{C_2}{20^2} - 5.38 \cdot 10^{-6} \ln 20 - 1.45 \cdot 10^{-6}$$

$$0 = C_1 \cdot 10 + \frac{C_2}{10} - 4.14 \cdot 10^{-6} \cdot 10 (\ln 10 - 0.5)$$

$$0 = 0.5 \cdot 10^2 C_1 + C_2 \ln 10 - 2.07 \cdot 10^{-6} \cdot 10^2 \ln 10 + 2.07 \cdot 10^{-6} 10^2 + C_3$$

$$1.3 \cdot C_1 - 1.45 \cdot 10^{-3} C_2 = 1.46 \cdot 10^{-5}$$

$$10 C_1 + 0.1 C_2 = 1.46 \cdot 10^{-5}$$

$$C_3 = -50 C_1 - 2.3 C_2 + 2.7 \cdot 10^{-4}$$

$$C_1 = \frac{1.46 \cdot 10^{-5} + 1.45 \cdot 10^{-3} C_2}{1.3}$$

$$10 \cdot \frac{1.46 \cdot 10^{-5} + 1.45 \cdot 10^{-3} C_2}{1.3} + 0.1 C_2 = 1.46 \cdot 10^{-5}$$

$$(1.35 \cdot 10^{-2} + 0.1) C_2 = 1.46 \cdot 10^{-5} - 1.35 \cdot 10^{-4}$$

$$C_2 = \frac{-6.04 \cdot 10^{-5}}{1.14 \cdot 10^{-1}} = -5.3 \cdot 10^{-4}$$

$$C_1 = \frac{1.46 \cdot 10^{-5} + 1.45 \cdot 10^{-3} (-5.3 \cdot 10^{-4})}{1.3} = 1.28 \cdot 10^{-5}$$

$$C_3 = -50 \cdot 1.28 \cdot 10^{-5} + (-5.3 \cdot 10^{-4}) \cdot (-2.3) + 2.7 \cdot 10^{-4} = 8.49 \cdot 10^{-4}$$

6. Řešení

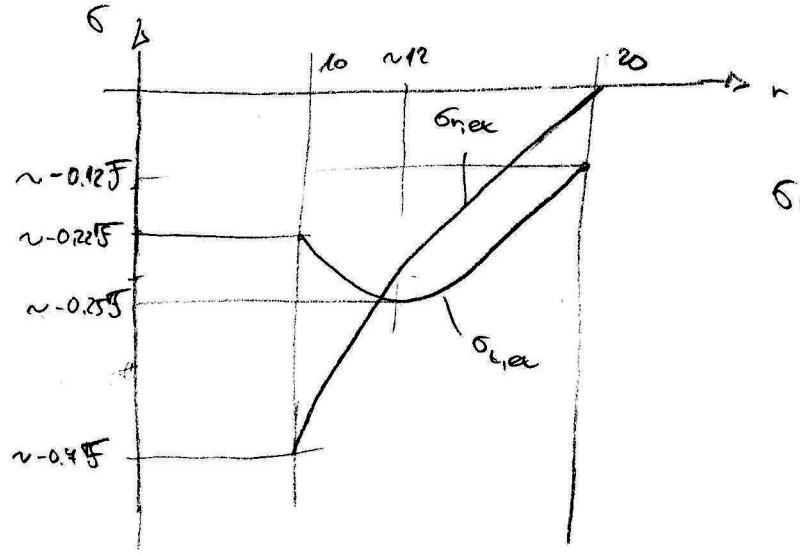
$$\sigma_{r,a} = \frac{6 M_r}{h^2} = 6 \cdot (-1.92 \cdot 10^{+4}) \left(1.3 \cdot 1.28 \cdot 10^{-5} - 0.7 \cdot (-5.3 \cdot 10^{-4}) \cdot \frac{1}{r^2} - 5.38 \cdot 10^{-6} \ln r - 1.45 \cdot 10^{-6} \right) = -1.15 \cdot 10^{5} \left(1.52 \cdot 10^{-5} + 3.41 \cdot 10^{-4} \frac{1}{r^2} - 5.38 \cdot 10^{-6} \ln r \right)$$

$$M_t = -B \left(\frac{v}{r} + \nu \frac{dv}{dr} \right) = -1.92 \cdot 10^{4} \left(C_1 + \frac{C_2}{r^2} - 4.14 \cdot 10^{-6} (\ln r - 0.5) + 0.3 \left[C_1 - \frac{C_2}{r^2} - 4.14 \cdot 10^{-6} (\ln r + 0.5) \right] \right) = -1.92 \cdot 10^{4} \left(1.3 \cdot 1.28 \cdot 10^{-5} + 0.7 \cdot (-5.3 \cdot 10^{-4}) \cdot \frac{1}{r^2} - 1.3 \cdot 4.14 \cdot 10^{-6} \ln r + 2.07 \cdot 10^{-6} \cdot 0.7 \right) = -1.92 \cdot 10^{4} \left(1.81 \cdot 10^{-5} - 3.41 \cdot 10^{-4} \frac{1}{r^2} - 5.38 \cdot 10^{-6} \ln r \right)$$

$$\sigma_{t,a} = \frac{6 M_t}{h^2} = -1.15 \cdot 10^{+5} \cdot \left(1.81 \cdot 10^{-5} - 3.41 \cdot 10^{-4} \frac{1}{r^2} - 5.38 \cdot 10^{-6} \ln r \right)$$

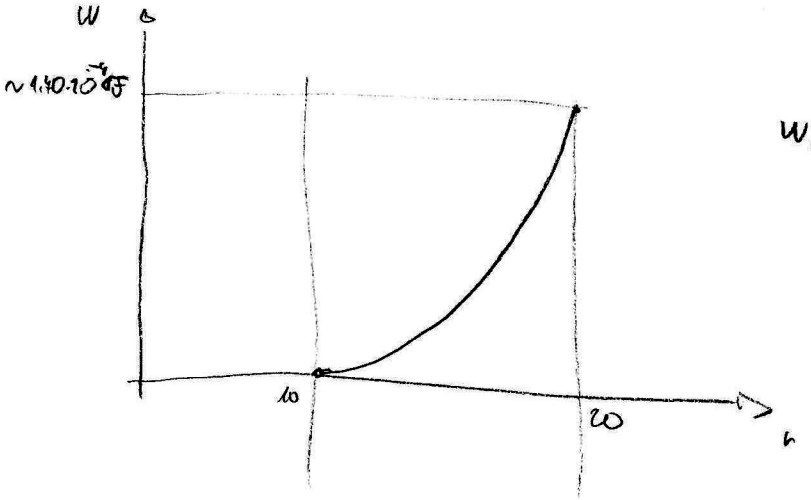
$$w = 0.5 \cdot 1.28 \cdot 10^{-5} r^2 + (-5.3 \cdot 10^{-4}) \ln r - 2.07 \cdot 10^{-6} r^2 \ln r + 2.07 \cdot 10^{-6} r^2 + 8.49 \cdot 10^{-4}$$

$$= 6.40 \cdot 10^{-6} r^2 - 5.3 \cdot 10^{-4} \ln r - 2.07 \cdot 10^{-6} r^2 \ln r + 2.07 \cdot 10^{-6} r^2 + 8.49 \cdot 10^{-4}$$



$$\sigma_{red} = |\sigma_{t,max} - \sigma_{r,max}| = |\sigma_{t,ek} - \sigma_{r,ek}|_{r=10} =$$

$$\cong 0{,}44\,F\ MPa$$



$$w_{max} = w|_{r=20} \cong 1{,}68 \cdot 10^{-4}\,F\ mm$$