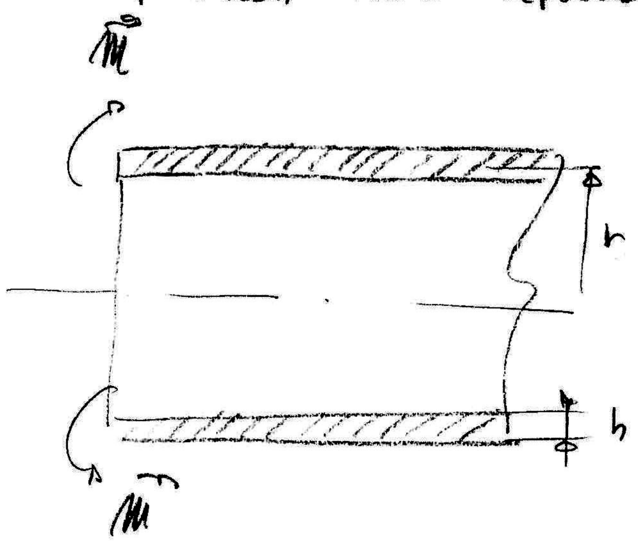
Př. k MS pružnosti určete bezpečnost a max. rozevření tažné skořepiny



mat: $\nu = 0.3$

$E = 2.1\cdot10^5\,\mathrm{MPa}$

geom: $r = 20\,\mathrm{mm}$

$h = 1\,\mathrm{mm}$

zatížení: $M \cdot 2\pi r = 1 \Rightarrow M = \frac{1}{2\pi\cdot 20} = 7.96\cdot10^{-3}\,\mathrm{N}$

$p = p_z = 0$

1. Partikulární řešení

$$u_p = \frac{r^2}{E}\left[p - \frac{\nu}{r}\left(C_0 - \int p_z\,dz\right)\right] = -\frac{r^2\nu}{Er}\cdot C_0 = \frac{r\nu}{E}\cdot C_0 = -\frac{20\cdot0.3}{2.1\cdot10^5}\cdot C_0 = -2.86\cdot10^{-5}$$

2. Natočení, momenty a ...

$$u = \tilde{u} + u_p = e^{-\beta z}\left(C_1 \sin\beta z + C_2\cos\beta z\right) + 2.86\cdot10^{-5} C_0$$

$$\beta = \sqrt[4]{\frac{3(1-\nu^2)}{r^2 h^2}} = \sqrt[4]{\frac{3(1-0.3^2)}{20^2\cdot 1^2}} = 2.87\cdot10^{-1}\,\frac{1}{\mathrm{mm}}, \quad B = \frac{Eh^3}{12(1-\nu^2)} = \frac{2.1\cdot10^5\cdot1^3}{12(1-0.3^2)} = 1.92\cdot10^4$$

$$u = e^{-2.87\cdot10^{-1} z}\left(C_1\sin(2.87\cdot10^{-1}z) + C_2\cos(2.87\cdot10^{-1}z)\right) + 2.86\cdot10^{-5}C_0$$

$$M_z = -2B\beta^2 e^{-\beta z}\left(-C_1\cos\beta z + C_2\sin\beta z\right) - 0 = -2\cdot1.92\cdot10^4\cdot(2.87\cdot10^{-1})^2\cdot$$

$$\cdot\, e^{-2.87\cdot10^{-1}z}\left(-C_1\cdot\cos(2.87\cdot10^{-1}\cdot z) + C_2\cdot\sin(2.87\cdot10^{-1}\cdot z)\right) =$$

$$= -3.16\cdot10^3\, e^{-2.87\cdot10^{-1}z}\left(-C_1\cos(2.87\cdot10^{-1}z) + C_2\sin(2.87\cdot10^{-1}z)\right)$$

$$T_{rz} = \frac{dM_z}{dz} = -2B\cdot\beta^3 e^{-\beta z}\left[C_1(\sin\beta z + \cos\beta z) + C_2(-\sin\beta z + \cos\beta z)\right] =$$

$$= -2\cdot1.92\cdot10^4\cdot(2.87\cdot10^{-1})^3 e^{-2.87\cdot10^{-1}z}\left[C_1\left(\sin(2.87\cdot10^{-1}z) + \cos(2.87\cdot10^{-1}z)\right)\right.$$

$$\left. + C_2\left(-\sin(2.87\cdot10^{-1}z) + \cos(2.87\cdot10^{-1}z)\right)\right] =$$

$$= -9.08\cdot10^2\, e^{-2.87\cdot10^{-1}z}\left[C_1\left(\sin(2.87\cdot10^{-1}z) + \cos(2.87\cdot10^{-1}z)\right) + \right.$$

$$\left. + C_2\left(-\sin(2.87\cdot10^{-1}z) + \cos(2.87\cdot10^{-1}z)\right)\right]$$

$$N_z = C_0 - \int p_z\,dz = C_0$$

3. Okr. podmínky:

$$M_z\big|_{z=0} = -M, \quad T_{rz}\big|_{z=0} = 0, \quad N_z\big|_{z=0} = 0$$

4. Dosazení C_0, C_1, C_2:

$$+3.16 \cdot 10^3 \cdot 1 \left(-C_1 + 0\right) = -\mathcal{M} = -7.96 \cdot 10^{-3}$$

$$-9.08 \cdot 10^2 \cdot \left[C_1 + C_2\right] = 0$$

$$C_0 = 0$$

$$C_1 = \frac{-7.96 \cdot 10^{-3}}{+3.16 \cdot 10^3} = -2.52 \cdot 10^{-6}$$

$$C_2 = -C_1 = +2.52 \cdot 10^{-6}$$

$$C_0 = 0$$

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5. Řešení

$$u = e^{-2.87 \cdot 10^{-1} z}\left(-2.52 \cdot 10^{-6} \sin\left(2.87 \cdot 10^{-1} z\right) + 2.52 \cdot 10^{-6} \cos\left(2.87 \cdot 10^{-1} z\right)\right)$$

$$u = e^{-2.87 \cdot 10^{-1} z}\, 2.52 \cdot 10^{-6}\left(-\sin\left(2.87 \cdot 10^{-1} z\right) + \cos\left(2.87 \cdot 10^{-1} z\right)\right)$$

$$u_z = 0$$

$$w = -\frac{\nu}{r}\int u\,dz = -\frac{\nu}{r}\int\left\{e^{-\beta z}\left(C_1 \sin\beta z + C_2 \cos\beta z\right) + 2.86 \cdot 10^{-5} C_0\right\} dz =$$

$$\int e^{-\beta z} \sin\beta z\, dz = \left\{\begin{array}{l} u = e^{-\beta z} \Rightarrow u' = -\beta e^{-\beta z} \\ v' = \sin\beta z \Rightarrow v = -\frac{1}{\beta}\cos\beta z \end{array}\right\} = e^{-\beta z} \cdot \left(-\frac{1}{\beta}\right)\cos\beta z - \int -\cancel{\beta} e^{-\beta z} \cdot$$

$$\cdot\left(-\frac{1}{\cancel{\beta}}\right)\cos\beta z\, dz = -\frac{1}{\beta} e^{-\beta z}\cos\beta z - \int e^{-\beta z}\cos\beta z\, dz = \left\{\begin{array}{l} u = e^{-\beta z} \Rightarrow u' = -\beta e^{-\beta z} \\ v' = \cos\beta z \Rightarrow v = \frac{1}{\beta}\sin\beta z \end{array}\right\} =$$

$$= -\frac{1}{\beta} e^{-\beta z}\cos\beta z - \left[e^{-\beta z} \cdot \frac{1}{\beta}\sin\beta z - \int -\cancel{\beta} e^{-\beta z}\frac{1}{\cancel{\beta}}\sin\beta z\right] = -\frac{1}{\beta} e^{-\beta z}\cos\beta z - \frac{1}{\beta} e^{-\beta z} \cdot$$

$$\cdot \sin\beta z - \int e^{-\beta z}\sin\beta z\, dz \Rightarrow 2\int e^{-\beta z}\sin\beta z\, dz = -\frac{1}{\beta} e^{-\beta z}\left(\cos\beta z + \sin\beta z\right)$$

$$\Rightarrow \int e^{-\beta z}\sin\beta z\, dz = -\frac{1}{2\beta} e^{-\beta z}\left(\cos\beta z + \sin\beta z\right)$$

$$\int e^{-\beta z}\cos\beta z\, dz = \left\{\begin{array}{l} u = e^{-\beta z} \Rightarrow u' = -\beta e^{-\beta z} \\ v' = \cos\beta z \Rightarrow v = \frac{1}{\beta}\sin\beta z \end{array}\right\} = e^{-\beta z} \cdot \frac{1}{\beta}\sin\beta z - \int -\cancel{\beta} e^{-\beta z} \cdot \frac{1}{\cancel{\beta}}\sin\beta z\, dz$$

$$= \frac{1}{\beta} e^{-\beta z}\sin\beta z + \int e^{-\beta z}\sin\beta z\, dz = \left\{\begin{array}{l} u = e^{-\beta z} \Rightarrow u' = -\beta e^{-\beta z} \\ v' = \sin\beta z \Rightarrow v = -\frac{1}{\beta}\cos\beta z \end{array}\right\} = \frac{1}{\beta} e^{-\beta z}\sin\beta z +$$

$$+ \left(-\frac{1}{\beta}\right) e^{-\beta z}\cos\beta z - \int -\cancel{\beta} e^{-\beta z}\left(-\frac{1}{\cancel{\beta}}\right)\cos\beta z\, dz = \frac{1}{\beta} e^{-\beta z}\left(\sin\beta z - \cos\beta z\right) - \int e^{-\beta z} \cdot$$

$$\cdot \cos\beta z\, dz \Rightarrow \int e^{-\beta z}\cos\beta z\, dz = -\frac{1}{2\beta} e^{-\beta z}\left(\cos\beta z - \sin\beta z\right)$$

$$= -\frac{\nu}{r}\left\{\left(-\frac{1}{2\beta}\right) e^{-\beta z}\left(\left(C_1 + C_2\right)\cos\beta z + \left(C_1 - C_2\right)\sin\beta z\right) + 2.86 \cdot 10^{-5} C_0 z\right\} =$$

$$= -\frac{0.3}{20}\left\{-\frac{1}{2\cdot 2.87\cdot 10^{-1}}\, e^{-2.87\cdot 10^{-1} z}\left[(-2.52\cdot 10^{-6}+2.52\cdot 10^{-6})\cos 2.87\cdot 10^{-1} z + (-2.52\cdot 10^{-6}-2.52\cdot 10^{-6})\cdot \sin 2.87\cdot 10^{-1} z\right]\right\} = -2.61\cdot 10^{-2}\cdot e^{-2.87\cdot 10^{-1} z}\left[5.04\cdot 10^{-6}\cdot \sin 2.87\cdot 10^{-1} z\right] = -1.32\cdot 10^{-7} e^{-2.87\cdot 10^{-1} z} \sin 2.87\cdot 10^{-1} z$$

$$\vartheta_{rz} = -2\cdot 1.92\cdot 10^{4}\,(2.87\cdot 10^{-1})^{3} e^{-2.87\cdot 10^{-1} z}\left[-2.52\cdot 10^{-6}\cdot(\sin 2.87\cdot 10^{-1} z + \cancel{\cos 2.87\cdot 10^{-1} z}) + (+2.52\cdot 10^{-6})(-\sin 2.87\cdot 10^{-1} z + \cancel{\cos 2.87\cdot 10^{-1} z})\right] = 9.08\cdot 10^{2}\cdot e^{-2.87\cdot 10^{-1} z}\cdot 2\cdot 2.52\cdot 10^{-6}\cdot \sin 2.87\cdot 10^{-1} z = 4.58\cdot 10^{-3} e^{-2.87\cdot 10^{-1} z} \sin 2.87\cdot 10^{-1} z$$

$$M_t = r\left(pr + \frac{d\vartheta_{rz}}{dz}\right) = 20\left(4.58\cdot 10^{-3}\,(-2.87\cdot 10^{-1})\, e^{-2.87\cdot 10^{-1} z} \sin 2.87\cdot 10^{-1} z + 4.58\cdot 10^{-3}\cdot e^{-2.87\cdot 10^{-1} z}\, 2.87\cdot 10^{-1}\cos 2.87\cdot 10^{-1} z\right) = 2.63\cdot 10^{-2}\, e^{-2.87\cdot 10^{-1} z}\left(-\sin 2.87\cdot 10^{-1} z + \cos 2.87\cdot 10^{-1} z\right)$$

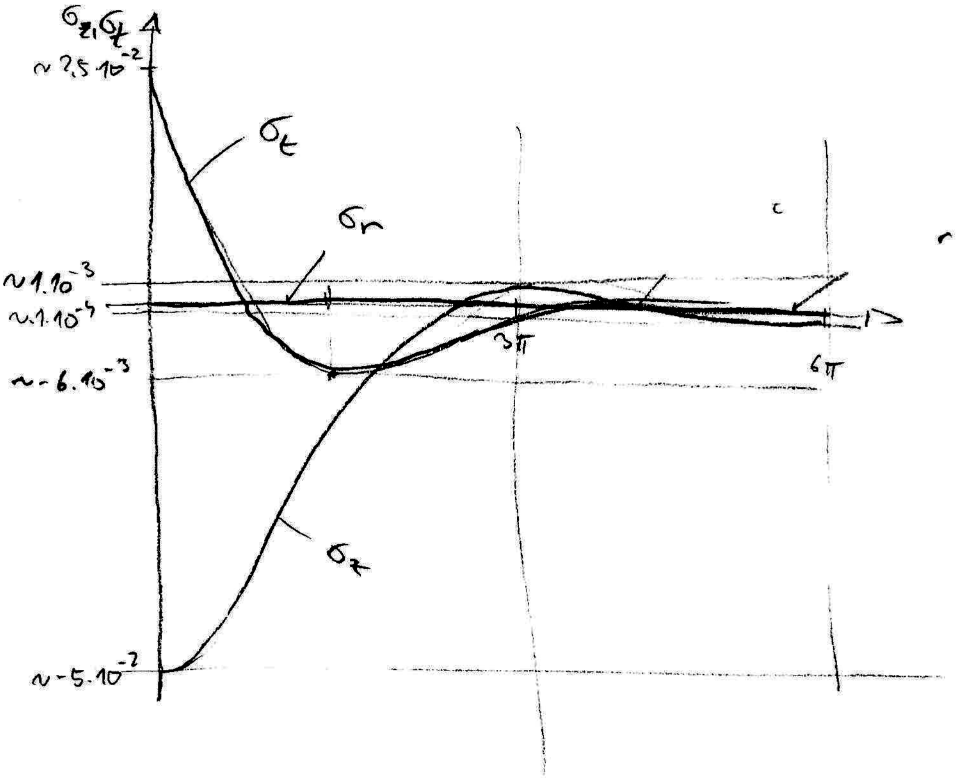
$$M_z = -3.16\cdot 10^{3} e^{-2.87\cdot 10^{-1} z}\left(+2.52\cdot 10^{-6}\cos(2.87\cdot 10^{-1} z) + 2.52\cdot 10^{-6}\sin(2.87\cdot 10^{-1} z)\right)$$
$$= +7.96\cdot 10^{-3} e^{-2.87\cdot 10^{-1} z}\left(-\cos 2.87\cdot 10^{-1} z - \sin 2.87\cdot 10^{-1} z\right)$$

$$M_t = 0.3\cdot M_z = +2.39\cdot 10^{-3} e^{-2.87\cdot 10^{-1} z}\left(-\cos 2.87\cdot 10^{-1} z - \sin 2.87\cdot 10^{-1} z\right)$$

$$\sigma_{z,ie} = \frac{N_z}{h} \pm \frac{6 M_z}{h^2} = \pm\frac{6}{1^2}\cdot(+7.96\cdot 10^{-3})\, e^{-2.87\cdot 10^{-1} z}\left(-\cos 2.87\cdot 10^{-1} z - \sin 2.87\cdot 10^{-1} z\right)$$
$$= \mp 4.78\cdot 10^{-2}\, e^{-2.87\cdot 10^{-1} z}\left(\cos 2.87\cdot 10^{-1} z + \sin 2.87\cdot 10^{-1} z\right)$$

$$\sigma_{t,ie} = \frac{N_t}{h} \pm \frac{6 M_t}{h^2} = \frac{1}{1}\cdot 2.63\cdot 10^{-2} e^{-2.87\cdot 10^{-1} z}\left(-\sin 2.87\cdot 10^{-1} z + \cos 2.87\cdot 10^{-1} z\right) \pm$$
$$\pm \frac{6}{1}\cdot(+2.39\cdot 10^{-3})\, e^{-2.87\cdot 10^{-1} z}\left(-\cos 2.87\cdot 10^{-1} z - \sin 2.87\cdot 10^{-1} z\right)$$
$$= e^{-2.87\cdot 10^{-1} z}\left[(2.63\cdot 10^{-2} \mp 2.39\cdot 10^{-3})\cos 2.87\cdot 10^{-1} z + (-2.63\cdot 10^{-2} \mp 2.39\cdot 10^{-3})\cdot \sin 2.87\cdot 10^{-1} z\right]$$

$$u_{max} = +2.52\cdot 10^{-6}\ \text{mm} = u\big|_{z=0}$$



$$\sigma_{red} = \max\left\{ |\sigma_r - \sigma_z|, |\sigma_r - \sigma_t|, |\sigma_z - \sigma_t| \right\} = |\sigma_z - \sigma_t|\Big|_{z=0} = $$

$$\cong \underline{\underline{7.5 \cdot 10^{-2}\ \mathrm{MPa}}}$$