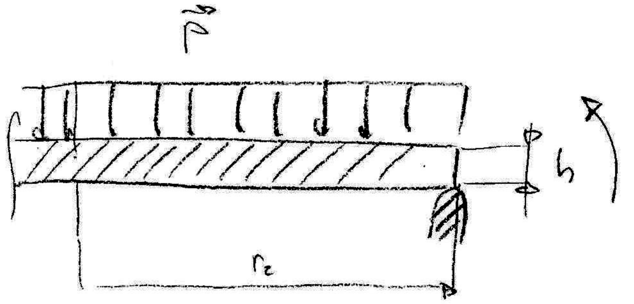
Př: Určete bezpečnost k MS a max. průhyb desky podle obr.



mat.: $\nu = 0.3$

$E = 2.1\cdot10^5$ MPa

geom: $r_2 = 20$ mm

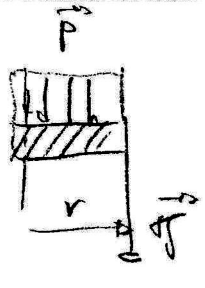
$h = 1$ mm

zatížení: $p\cdot\pi r_2^2 = 1 \Rightarrow p = \frac{1}{\pi\cdot 20^2} = 7.96\cdot10^{-4}\ \frac{N}{mm}$

$M\cdot 2\pi r_2 = 1 \Rightarrow M = \frac{1}{2\pi 20} = 7.96\cdot10^{-3}$ N

tuhost desky: $B = \frac{E\cdot h^3}{12(1-\nu^2)} = \frac{2.1\cdot10^5\cdot 1^3}{12(1-0.3^2)} = 1.92\cdot10^4$ Nmm

1. Uvolnění elementu



$$2\cancel{\pi} r\cdot T = -\cancel{\pi} r^2 p$$

$$T = -\tfrac{1}{2} r p = -\tfrac{1}{2} r\cdot 7.96\cdot10^{-4} \Rightarrow T = -3.98\cdot10^{-4}\, r$$

2. Partikulární řešení

$$\vartheta_p = -\frac{1}{rB}\int r\int T\,dr\,dr = -\frac{1}{1.92\cdot10^4}\cdot\frac{1}{r}\int r\int -3.98\cdot10^{-4}\, r\,dr\,dr = 2.07\cdot10^{-8}\,\frac{1}{r}\int r\,\frac{r^2}{2}\,dr =$$

$$= 2.07\cdot10^{-8}\,\frac{1}{\cancel{r}}\cdot\frac{\cancel{r^4}\,r^3}{8} = 2.59\cdot10^{-9}\, r^3$$

3. Natočení, momenty, průhyb

$$\vartheta = C_1\cdot r + \frac{C_2}{r} + 2.59\cdot10^{-9} r^3$$

$$\frac{d\vartheta}{dr} = C_1 - \frac{C_2}{r^2} + 2.59\cdot10^{-9}\, 3\cdot r^2 = C_1 - \frac{C_2}{r^2} + 7.77\cdot10^{-9} r^2$$

$$\frac{\vartheta}{r} = C_1 + \frac{C_2}{r^2} + 2.59\cdot10^{-9}\cdot r^2$$

$$M_r = -B\left(\frac{d\vartheta}{dr} + \nu\frac{\vartheta}{r}\right) = -1.92\cdot10^4\cdot\left(C_1 - \frac{C_2}{r^2} + 7.77\cdot10^{-9} r^2 + 0.3\left[C_1 + \frac{C_2}{r^2} + 2.59\cdot10^{-9} r^2\right]\right) =$$

$$= -1.92\cdot10^4\left(1.3C_1 - 0.7\frac{1}{r^2}C_2 + 8.55\cdot10^{-9} r^2\right)$$

$$w = \int\vartheta\,dr + C_3 = \frac{1}{2}C_1 r^2 + C_2\ln r + 2.59\cdot10^{-9}\frac{r^4}{4} + C_3 = 0.5r^2 C_1 + C_2\ln r + 6.48\cdot10^{-10} r^4 + C_3$$

4. Okr. podmínky

$$\vartheta\big|_{r=0} = 0,\quad M_r\big|_{r=r_2} = M,\quad w\big|_{r=r_2} = 0$$

5. Stanovení C_1, C_2, C_3

$$0 = C_1 r + \frac{C_2}{r} + 2.59\cdot10^{-9}\cdot r^3 \quad \text{pro } r \to 0$$

$$4.96\cdot10^{-4} = -1.92\cdot10^{4}\left(1.3\cdot C_1 - 0.7\frac{1}{20^2}C_2 + 8.55\cdot10^{-9}\cdot 20^2\right)$$

$$0 = 0.5\cdot 20^2\cdot C_1 + C_2\cdot \ln 20 + 6.48\cdot10^{-10}\,20^4 + C_3$$

$$\boxed{C_2 = 0}$$

$$-4.15\cdot10^{-8} = 1.3\,C_1 + 3.42\cdot10^{-6} \Rightarrow C_1 = \frac{-4.15\cdot10^{-8} - 3.42\cdot10^{-6}}{1.3} = -1.43\cdot10^{-6}$$

$$C_3 = -0.5\cdot 20^2\cdot(-1.43\cdot10^{-6}) - 6.48\cdot10^{-10}\cdot 20^4 = 3.46\cdot10^{-4}$$

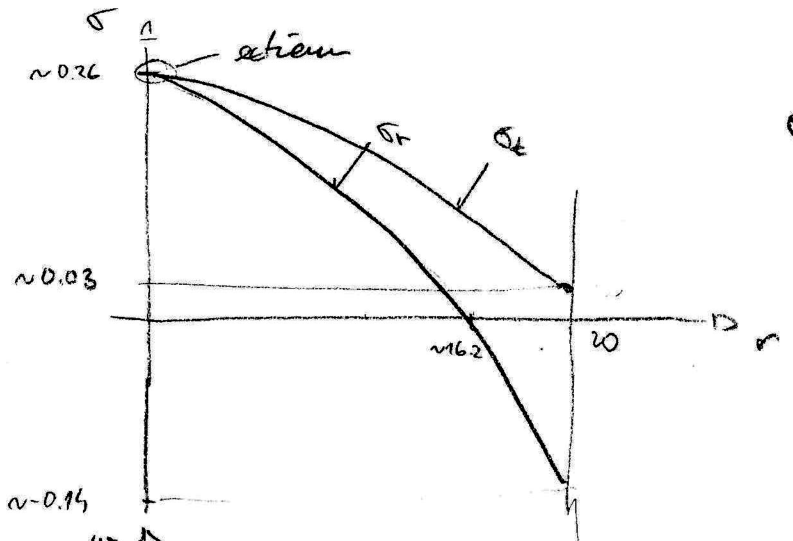
6. Řešení

$$\sigma_{r,ex} = \frac{6M_r}{h^2} = 6\cdot(-1.92\cdot10^{4})\left(1.3\cdot(-1.43\cdot10^{-6}) + 8.55\cdot10^{-9}r^2\right) = -1.15\cdot10^{5}\cdot\left(-2.25\cdot10 + 8.55\cdot10^{-9}\cdot r^2\right)$$

$$M_t = -B\left(\frac{\vartheta}{r} + \nu\frac{d\vartheta}{dr}\right) = -1.92\cdot10^{4}\left(C_1 + 2.59\cdot10^{-9}r^2 + 0.3\left[C_1 + 7.77\cdot10^{-9}r^2\right]\right) = -1.92\cdot10^{4}\left(1.3C_1 + 4.92\cdot10^{-9}r^2\right)$$

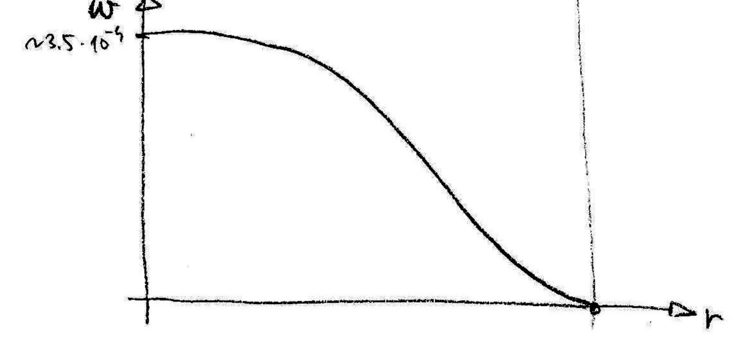
$$\sigma_{t,ex} = \frac{6M_t}{h^2} = 6\cdot(-1.92\cdot10^{4})\left(1.3\cdot(-1.43\cdot10^{-6}) + 4.92\cdot10^{-9}\cdot r^2\right) = -1.15\cdot10^{5}\left(-2.25\cdot10^{-6} + 4.92\cdot10^{-9}r^2\right)$$

$$w = 0.5\cdot r^2\cdot(-1.43\cdot10^{-6}) + 6.48\cdot10^{-10}r^4 + 3.46\cdot10^{-4} = -8.65\cdot10^{-7}r^2 + 6.48\cdot10^{-10}\cdot r^4 + 3.46\cdot10$$



$$\sigma_{red} = \left|\sigma_r - \sigma_t\right|\Big|_{r=20} = \left|-0.14 - 0.03\right| = 0.14\ \text{MPa}$$

$$k = \frac{\sigma_k}{\sigma_{red}}$$



$$w\big|_{r=0} = 3.46\cdot10^{-4}\ \text{mm}$$