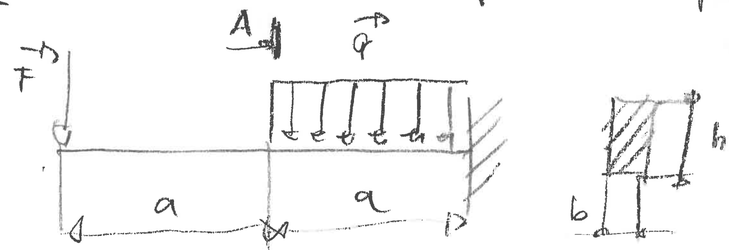
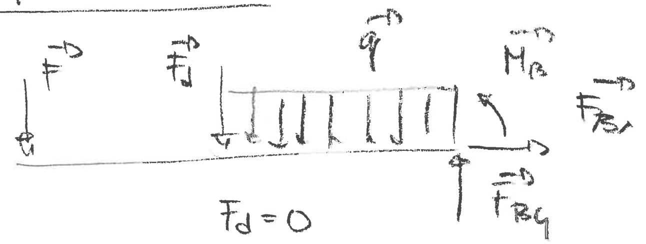
1. Určete maximální napětí σ a průhyb volného konce v místě A.



1. Úplné uvolnění



$F_d = 0$

2. Stat. rozbor

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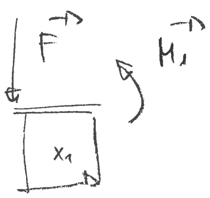
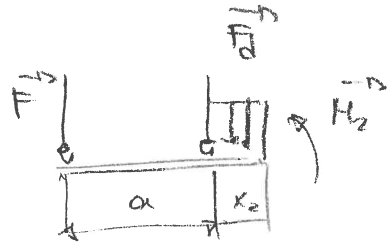
3. VVÚ



$x_1 \in (0, a)$

$M_1 = -F \cdot x_1$

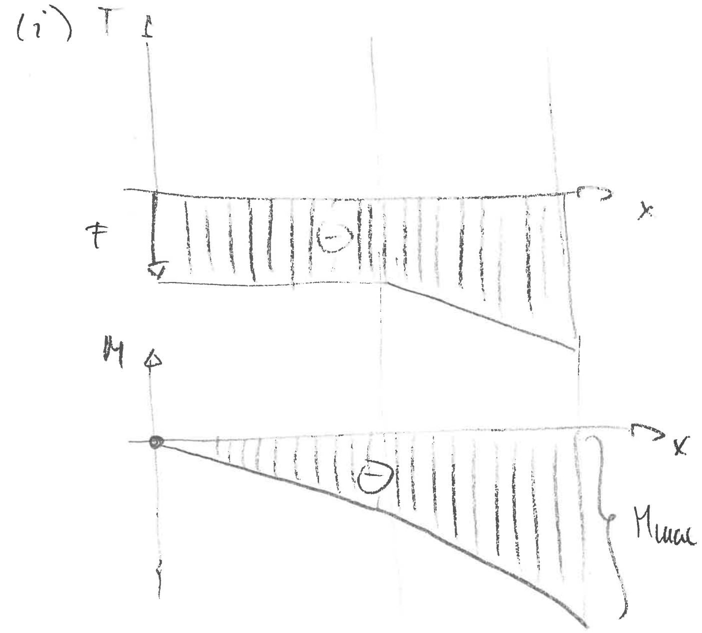


$x_2 \in (0, a)$

$M_2 = -F \cdot (x_2 + a) - F_d \cdot x_2 - q\frac{x_2^2}{2}$

4. Řešení

(i)



$M_{max} = -2Fa - q\frac{a^2}{2}$

$W_o = \frac{1}{6} b \cdot h^2$

$\sigma_{max} = \frac{M_{max}}{W_o} = \frac{-4F \cdot a - qa^2}{\frac{1}{3} b \cdot h^2} = \frac{-12F \cdot a - 3qa^2}{b h^2}$

(ii)

$$w_F = \frac{\partial W}{\partial F} = \frac{1}{EJ}\int_0^a M_1 \frac{\partial M_1}{\partial F} dx_1 + \frac{1}{EJ}\int_0^a M_2 \frac{\partial M_2}{\partial F} dx_2 = \frac{1}{EJ}\left\{\int_0^a -F \cdot x_1 \cdot (-x_1) dx_1 + \int_0^a \left(-F(x_2 + a) - \frac{q x_2^2}{2}\right) \cdot (-1) \cdot (x_2 + a) dx_2\right\} = \frac{1}{EJ}\left\{F\frac{a^3}{3} + \int_0^a F(x_2^2 + 2x_2 a + a^2) + \frac{q}{2}(x_2^3 + a x_2^2) dx_2\right\} = \frac{1}{EJ}\left\{F\frac{a^3}{3} + F\left(\frac{a^3}{3} + 2a\frac{a^2}{2} + a^3\right) + \frac{q}{2}\left(\frac{a^4}{4} + a\frac{a^3}{3}\right)\right\} = \frac{1}{EJ}\left\{Fa^3\left(\frac{2}{3} + 2\right) + \frac{qa^4}{2}\left(\frac{1}{4} + \frac{1}{3}\right)\right\}$$

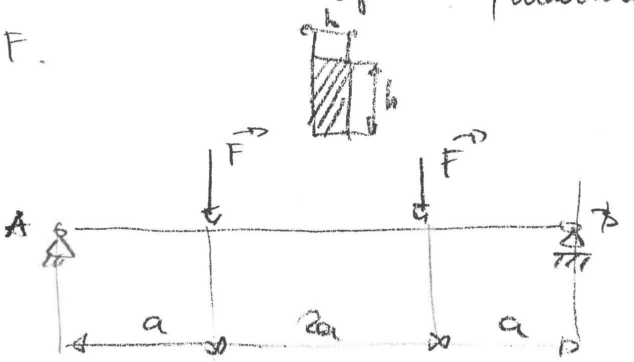
$$= \frac{1}{EJ}\left\{\frac{8}{3}Fa^3 + \frac{7}{24}qa^4\right\}$$

(iii) $u_A = \frac{\partial W}{\partial F_d} = \frac{1}{EJ}\left\{\int_0^a M_1 \frac{\partial M_1}{\partial F_d} dx_1 + \int_0^a M_2 \frac{dM_2}{dF_d} dx_2\right\}_0 = \frac{1}{EJ}\int_0^a \left(-F\cdot(x_2+a) - q\frac{x_2^2}{2}\right)\cdot(-x_2)\, dx_2 =$

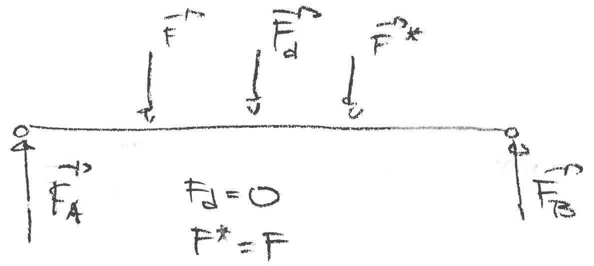
$= \frac{1}{EJ}\int_0^a \left[F(x_2^2+ax_2)+q\frac{x_2^3}{2}\right]dx_2 = \frac{1}{EJ}\left\{F\left(\frac{a^3}{3}+a\frac{a^2}{2}\right) + \frac{q}{2}\frac{a^4}{4}\right\} =$

$= \frac{1}{EJ}\left\{Fa^3\left(\frac{1}{3}+\frac{1}{2}\right)+\frac{q}{8}a^4\right\} = \frac{1}{EJ}\left\{\frac{5}{6}Fa^3+\frac{1}{8}qa^4\right\}$

Př: Určete maximální napětí σ, maximální průhyb a průhyb v místě působení síly F.



1. Úplné uvolnění



$F_d = 0$
$F^* = F$

$\Sigma M_A: F_B\cdot 4a - F^* 3a - F_d\cdot 2a - F\cdot a = 0 \Rightarrow F_B = \frac{3F^*+2F_d+F}{4}$

$\Sigma M_B: F_A\cdot 4a - F\,3a - F_d\,2a - F^* a = 0 \Rightarrow F_A = \frac{F^*+2F_d+3F}{4}$

2. Stat. rozbor

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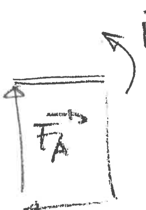
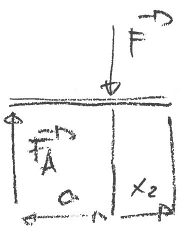
3. VVÚ



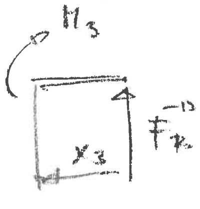
$x_1 \in (0,a)$

$M_1 = F_A\cdot x_1 = \frac{3F+2F_d+F^*}{4}\cdot x_1$



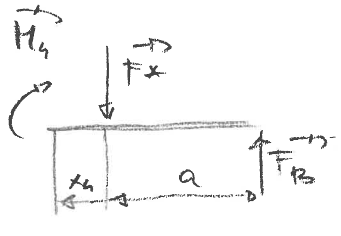
$x_2 \in (0,a)$

$M_2 = F_A(a+x_2) - F x_2 = \frac{3F+2F_d+F^*}{4}(a+x_2) - F x_2$



$x_3 \in (0,a)$

$M_3 = F_B\cdot x_3 = \frac{3F^*+2F_d+F}{4}\cdot x_3$

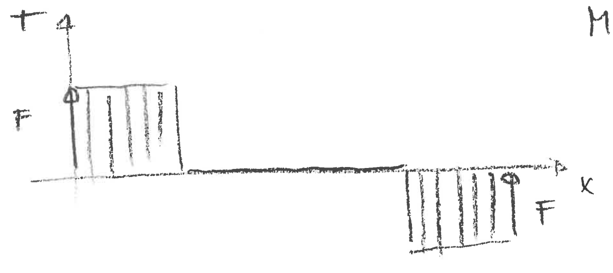


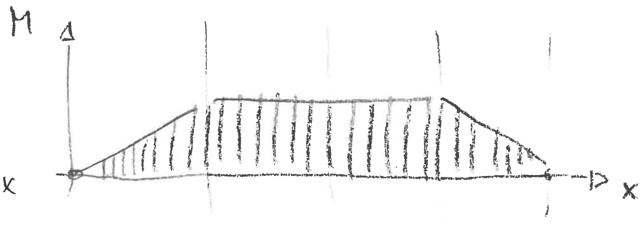
$x_4 \in (0,a)$

$M_4 = F_B(x_4+a) - F^* x_4 = \frac{3F^*+2F_d+F}{4}(x_4+a) - F^* x_4$

4. Řešení

(i)



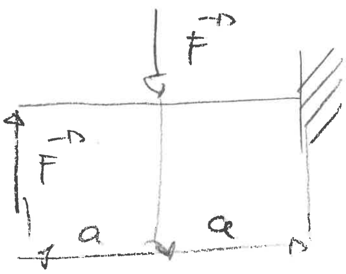


$$M_{omax} = F\cdot 2a - F\cdot a = F\cdot a \quad , \; W_o = \frac{1}{6} b h^2 \implies \sigma_{max} = \frac{M_o}{W_o} = \frac{F a}{\frac{1}{6} b h^2} = \frac{6 F a}{b\cdot h^2}$$

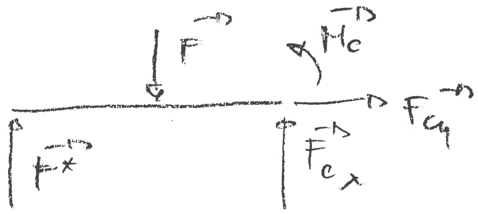
(ii)
$$w_F = \frac{\partial W}{\partial F} = \frac{\partial W}{\partial F^*} = \int_0^a \frac{M_1}{EJ}\frac{\partial M_1}{\partial F}dx_1 + \int_0^a \frac{M_2}{EJ}\frac{\partial M_2}{\partial F}dx_2 + \int_0^a \frac{M_3}{EJ}\frac{\partial M_3}{\partial F}dx_3 +$$
$$+ \int_0^a \frac{M_4}{EJ}\frac{\partial M_4}{\partial F}dx_4 = \frac{1}{EJ}\left\{\int_0^a \left[F\cdot x_1 \cdot \frac{3}{4}x_1 + \left[F(a+\cancel{x_1}) - F\cancel{x_1}\right]\cdot\left[\frac{3}{4}(a+x_1) - x_1\right]\right.\right.$$
$$\left.\left. + F\cdot x_1 \frac{1}{4}x_1 + \left[F(\cancel{x_1}+a) - F\cdot\cancel{x_1}\right]\cdot \frac{1}{4}(x_1+a)\right]dx_1\right\} = \frac{F}{EJ}\left\{\int_0^a \left[\frac{3}{4}x_1^2 + a\left(\frac{3}{4}a - \cancel{\frac{1}{4}x_1}\right)\right.\right.$$
$$\left.\left. + \frac{1}{4}x_1^2 + \frac{1}{4}a(\cancel{x_1}+a)\right]dx_1\right\} = \frac{F}{EJ}\left\{\int_0^a \left[x_1^2 + a^2\right]dx_1\right\} = \frac{F}{EJ}\left\{\frac{a^3}{3} + a^3\right\} =$$
$$= \frac{4F\cdot a^3}{3EJ}$$

(iii)
$$w_{F_d} = \frac{\partial W}{\partial F_d} = \int_0^a \frac{M_1}{EJ}\frac{\partial M_1}{\partial F_d}dx_1 + \int_0^a \frac{M_2}{EJ}\frac{\partial M_2}{\partial F_d}dx_2 + \int_0^a \frac{M_3}{EJ}\frac{\partial M_3}{\partial F_d}dx_3 + \int_0^a \frac{M_4}{EJ}\frac{\partial M_4}{\partial F_d}dx_4 =$$
$$= \frac{1}{EJ}\left\{\int_0^a \left[F\cdot x_1 \cdot \frac{1}{2}x_1 + \left[F(a+\cancel{x_1}) - F\cancel{x_1}\right]\cdot\frac{1}{2}(a+x_1) + F\cdot x_3\cdot\frac{1}{2}x_3 + \left[F(\cancel{x_1}+a) - F\cancel{x_1}\right]\right.\right.$$
$$\left.\left.\cdot\frac{1}{2}(x_1+a)\right]dx_1\right\} = \frac{F}{EJ}\left\{\int_0^a\left[\frac{1}{2}x_1^2 + \frac{1}{2}a^2 + \frac{1}{2}a x_1 + \frac{1}{2}x_1^2 + \frac{1}{2}a x_1 + \frac{1}{2}a^2\right]dx_1\right\} =$$
$$= \frac{F}{EJ}\left\{\int_0^a\left[x_1^2 + a x_1 + a^2\right]dx_1\right\} = \frac{F}{EJ}\left\{\left[\frac{a^3}{3} + a\frac{a^2}{2} + a^3\right]\right\} = \frac{11 F\cdot a^3}{6EJ} = w_{max}$$

Poznamka (využití symetrie, lepší způsob řešení)



1. Úplné uvolnění



$F^* = F$

2. Stat. rozbor

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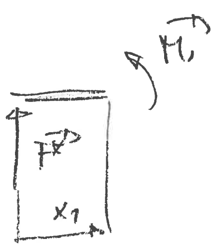
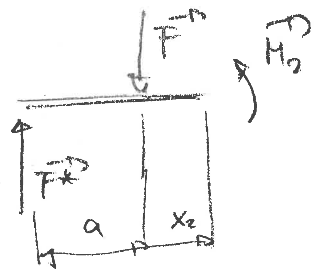
3. VVÚ



$x_1 \in (0,a)$

$M_1 = F^* x_1$

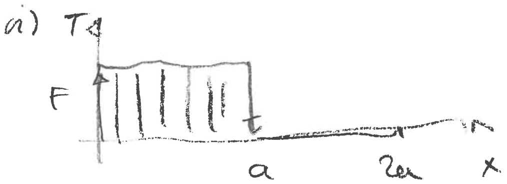


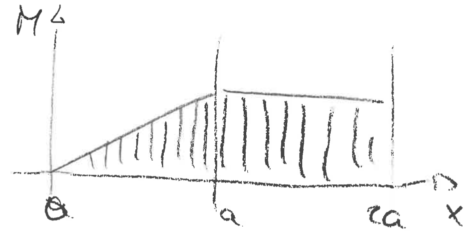
$x_2 \in (0,a)$

$M_2 = F^*(\cancel{x_2}+a) - F\cancel{x_2} = F\cdot a$

4. Řešení

a)





$M_{omax} = F\cdot a$

$W_o = \frac{1}{6} b\cdot h^2$

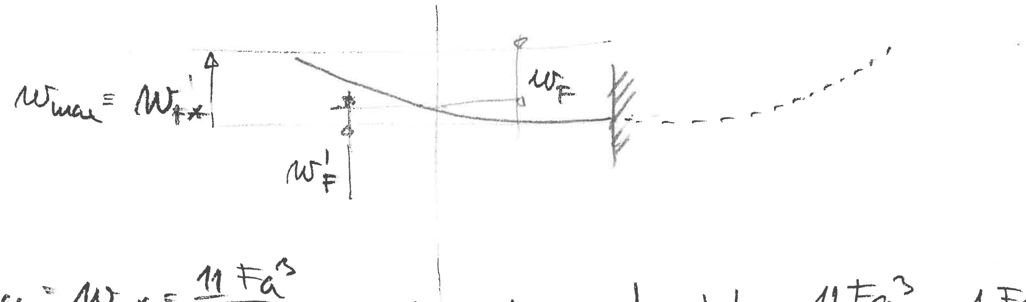
$\sigma_{max} = \frac{6 F a}{b\cdot h^2}$

(i) $w_F' = \frac{\partial W}{\partial F} = \int_0^a \frac{M_1}{EJ}\frac{\partial M_1}{\partial F}dx_1 + \int_0^a \frac{M_2}{EJ}\frac{\partial M_2}{\partial F}dx_2 = \frac{1}{EJ}\int_0^a F\cdot a\cdot(-x_2)\,dx_2 =$

$= \frac{-Fa}{EJ}\frac{a^2}{2} = -\frac{Fa^3}{2EJ}$

(ii) $w_{F^*} = \frac{\partial W}{\partial F^*} = \int_0^a \frac{M_1}{EJ}\frac{\partial M_1}{\partial F^*}dx_1 + \int_0^a \frac{M_2}{EJ}\frac{\partial M_2}{\partial F^*}dx_2 = \frac{1}{EJ}\int_0^a F\cdot 1\cdot x_1\,dx_1 + \frac{1}{EJ}\int_0^a Fa\cdot$

$\cdot(x_2+a)\,dx_2 = \frac{F}{EJ}\left\{\frac{a^3}{3}+\frac{a^3}{2}+a^3\right\} = \frac{11F\cdot a^3}{6EJ}$



$w_{max} = w_{F^*} = \frac{11}{6}\frac{Fa^3}{EJ}$, $w_F = w_{max} - |w_F'| = \frac{11Fa^3}{6EJ} - \frac{1}{2}\frac{Fa^3}{EJ} = \frac{Fa^3}{EJ}\left\{\frac{11}{6}-\frac{1}{2}\right\} =$

$= \frac{Fa^3}{EJ}\frac{8}{6} = \frac{Fa^3}{EJ}1\frac{1}{3} = \frac{Fa^3}{EJ}\frac{4}{3}$