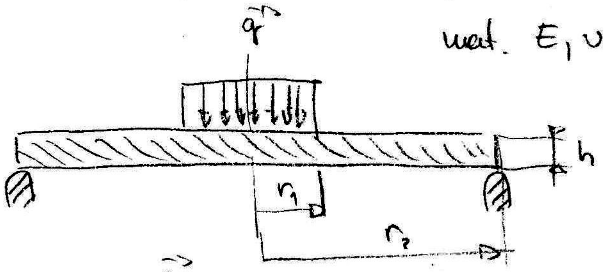
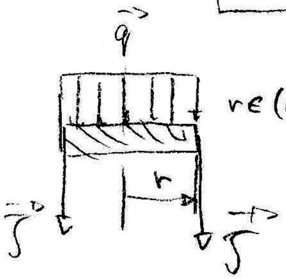
Př. Určete bezpečnost k MS pružnosti

mat. E, ν, σ_k



$$\int r \ln r \, dr = \left\{ \begin{array}{l} u = \ln r \Rightarrow u' = \frac{1}{r} \\ v' = r \Rightarrow v = \frac{1}{2} r^2 \end{array} \right\} = \frac{1}{2} r^2 \ln r - \int \frac{1}{r} \cdot \frac{1}{2} r^2 dr = \frac{1}{2} r^2 \ln r - \frac{1}{4} r^2 = \frac{1}{2} r^2 \left(\ln r - \frac{1}{2}\right)$$

$$\left(\frac{1}{2} r^2 \left(\ln r - \frac{1}{2}\right)\right)' = r\left(\ln r - \frac{1}{2}\right) + \frac{1}{2} r^2 \cdot \frac{1}{r} = r \ln r$$



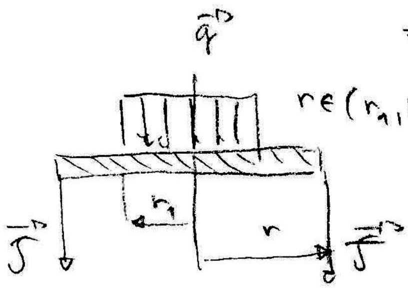
$r \in (0, r_1)$

$$2\pi r T_1 = -\pi r^2 q$$

$$T_1 = -\frac{1}{2} q r \Rightarrow \vartheta_{p1} = -\frac{1}{rB} \int r \int \left(-\frac{1}{2} q r\right) dr\, dr =$$

$$= -\frac{1}{rB} \int r \left(-\frac{1}{2} q\right) \frac{r^2}{2} dr = +\frac{1}{rB} \frac{1}{4} q \int r^3 dr = \frac{q}{4Br} \frac{r^4}{4} =$$

$$= \frac{1}{16} \frac{q r^3}{B}$$



$r \in (r_1, r_2)$

$$2\pi r T_2 = -\pi r_1^2 q$$

$$T_2 = -\frac{q r_1^2}{2r} \Rightarrow \vartheta_{p2} = -\frac{1}{rB} \int r \int \left(-\frac{1}{2} q r_1^2 \frac{1}{r}\right) dr\, dr =$$

$$= -\frac{1}{rB} \int r \left(-\frac{1}{2} q r_1^2\right) \ln r \, dr = \frac{q r_1^2}{2rB} \cdot \frac{1}{2} r^2 \left(\ln r - \frac{1}{2}\right) =$$

$$= \frac{q r_1^2}{4B} r \left(\ln r - \frac{1}{2}\right)$$

$$\vartheta_1 = C_1^1 r + \vartheta_{p1} = C_1^1 r + \frac{q}{16B} r^3$$

$$w_1 = \int \vartheta_1 dr + C_3^1 = \int \left(C_1^1 r + \frac{q}{16B} r^3\right) dr + C_3^1 = C_1^1 \frac{1}{2} r^2 + \frac{q}{16B} \frac{r^4}{4} + C_3^1 = \frac{1}{2} C_1^1 r^2 + \frac{q}{64B} r^4 + C_3^1$$

$$\vartheta_2 = C_1^2 r + C_2^2 \frac{1}{r} + \vartheta_{p2} = C_1^2 r + C_2^2 \frac{1}{r} + \frac{q r_1^2}{4B} r \left(\ln r - \frac{1}{2}\right)$$

$$w_2 = \int \vartheta_2 dr + C_3^2 = \int \left(C_1^2 r + C_2^2 \frac{1}{r} + \frac{q r_1^2}{4B} r \left(\ln r - \frac{1}{2}\right)\right) dr + C_3^2 = \frac{1}{2} C_1^2 r^2 + C_2^2 \ln r +$$

$$+ \frac{q r_1^2}{4B} \left[\frac{1}{2} r^2 \left(\ln r - \frac{1}{2}\right) - \frac{1}{4} r^2\right] + C_3^2 = \frac{1}{2} C_1^2 r^2 + C_2^2 \ln r + \frac{q r_1^2}{4B} \frac{1}{2} r^2 (\ln r - 1) + C_3^2$$

$$\frac{d\vartheta_1}{dr} = C_1^1 + \frac{3q}{16B} r^2$$

$$\frac{d\vartheta_2}{dr} = C_1^2 - C_2^2 \frac{1}{r^2} + \frac{q r_1^2}{4B} \left[\ln r - \frac{1}{2} + \cancel{r} \cdot \frac{1}{r}\right] = C_1^2 - C_2^2 \frac{1}{r^2} + \frac{q r_1^2}{4B} \left(\ln r + \frac{1}{2}\right)$$

$$M_{r1} = -B \left(\frac{d\vartheta_1}{dr} + \nu \frac{\vartheta_1}{r}\right) = -B \left(C_1^1 + \frac{3q}{16B} r^2 + \nu \left(C_1^1 + \frac{q}{16B} r^2\right)\right) = -B \left[C_1^1 (1+\nu) + \right.$$

$$\left. + \frac{q}{16B} r^2 (3+\nu)\right]$$

$$M_{r2} = -B \left(\frac{d\vartheta_2}{dr} + \nu \frac{\vartheta_2}{r}\right) = -B \left(C_1^2 - C_2^2 \frac{1}{r^2} + \frac{q r_1^2}{4B} \left(\ln r + \frac{1}{2}\right) + \nu \left(C_1^2 + C_2^2 \frac{1}{r^2} + \frac{q r_1^2}{4B} \left(\ln r - \frac{1}{2}\right)\right)\right)$$

$$= -B \left[C_1^2 (1+\nu) + C_2^2 (-1+\nu) \frac{1}{r^2} + \frac{q r_1^2}{4B} (1+\nu) \ln r + \frac{q r_1^2}{8B} (1-\nu)\right]$$

okr. podm.: $w_2\big|_{r=r_2}=0$ podm. komp.: $w_1\big|_{r=r_1}=w_2\big|_{r=r_1}$

$M_{r2}\big|_{r=r_2}=0$ $\vartheta_1\big|_{r=r_1}=\vartheta_2\big|_{r=r_1}$

$M_{r1}\big|_{r=r_1}=M_{r2}\big|_{r=r_1}$

$$M_{r_2}\big|_{r=r_2}=-B\left[C_1^2(1+\nu)+C_2^2(-1+\nu)\frac{1}{r_2^2}+\frac{qr_1^2}{4B}(1+\nu)\ln r_2+\frac{qr_1^2}{8B}(1-\nu)\right]=0$$

$$\vartheta_1\big|_{r=r_1}=C_1^1 r_1+\frac{q}{16B}r_1^3=C_1^2 r_1+C_2^2\frac{1}{r_1}+\frac{qr_1^2}{4B}r_1\left(\ln r_1-\frac{1}{2}\right)=\vartheta_2\big|_{r=r_1}$$

$$M_{r_1}\big|_{r=r_1}=-B\left[C_1^1(1+\nu)+\frac{q}{16B}r_1^2(3+\nu)\right]=-B\left[C_1^2(1+\nu)+C_2^2(-1+\nu)\frac{1}{r_1^2}+\frac{qr_1^2}{4B}(1+\nu)\ln r_1+\frac{qr_1^2}{8B}(1-\nu)\right]=M_{r2}\big|_{r=r_1}$$

$$C_1^1 r_1^2-C_1^2 r_1^2-C_2^2=\overbrace{-\frac{q}{16B}r_1^4+\frac{q}{4B}r_1^4\left(\ln r_1-\frac{1}{2}\right)}^{b_1}$$

$$C_1^2 r_2^2(1+\nu)-C_2^2(1-\nu)=\overbrace{-\frac{q}{4B}r_1^2r_2^2(1+\nu)\ln r_2+\frac{q}{8B}r_1^2r_2^2(1-\nu)}^{b_2}$$

$$C_1^1 r_1^2(1+\nu)-C_1^2 r_1^2(1+\nu)+C_2^2(1-\nu)=\underbrace{-\frac{q}{16B}r_1^4(3+\nu)+\frac{q}{4B}r_1^4(1+\nu)\ln r_1+\frac{q}{8B}r_1^4\cdot(1-\nu)}_{b_3}$$

$$C_1^1 r_1^2-C_1^2 r_1^2-C_2^2=b_1$$

$$C_1^2 r_2^2(1-\nu)-C_2^2(1-\nu)=b_2 \Rightarrow C_1^2=\frac{1}{r_2^2(1-\nu)}\left[b_2+C_2^2(1-\nu)\right]$$

$$C_1^1 r_1^2(1+\nu)-C_1^2 r_1^2(1+\nu)+C_2^2(1-\nu)=b_3$$

$$C_1^1 r_1^2-\frac{1}{r_2^2(1-\nu)}\left[b_2+C_2^2(1-\nu)\right]\cdot r_1^2-C_2^2=b_1$$

$$C_1^1 r_1^2(1+\nu)-\frac{1}{r_2^2(1-\nu)}\left[b_2+C_2^2(1-\nu)\right]r_1^2(1+\nu)+C_2^2(1-\nu)=b_3$$

$$\Rightarrow C_1^1=\frac{1}{r_1^2}b_1+\frac{1}{r_2^2(1-\nu)}b_2+C_2^2\left[\frac{1}{r_1^2}+\frac{1}{r_2^2}\right]$$

$$\left[\frac{1}{r_1^2}b_1+\frac{1}{r_2^2(1-\nu)}b_2+C_2^2\left[\frac{1}{r_1^2}+\frac{1}{r_2^2}\right]\right]r_1^2(1+\nu)-\frac{r_1^2(1+\nu)}{r_2^2(1-\nu)}b_2-C_2^2\frac{r_1^2(1+\nu)}{r_2^2}+$$

$$+C_2^2(1-\nu)=b_3 \Rightarrow C_2^2\left[(1+\cancel{\nu})+\frac{r_1^2}{r_2^2}\cancel{(1+\nu)}-\frac{r_1^2}{r_2^2}\cancel{(1+\nu)}+(1-\cancel{\nu})\right]=$$

$$=-(1+\nu)b_1-\cancel{\frac{r_1^2(1+\nu)}{r_2^2(1-\nu)}}b_2+\cancel{\frac{r_1^2(1+\nu)}{r_2^2(1-\nu)}}b_2+b_3 \Rightarrow$$

$$\Rightarrow \boxed{C_2^2=-(1+\nu)b_1+b_3}$$

$$C_1^1 = \cancel{\frac{1}{r_1^2} b_1} + \frac{1}{r_2^2(1-\nu)} b_2 + \left(-(\cancel{1}+\nu) b_1 + b_3\right)\left(\frac{1}{r_1^2} + \frac{1}{r_2^2}\right) \Rightarrow$$

$$\Rightarrow \boxed{C_1^1 = \left(-\frac{\nu}{r_1^2} - \frac{1+\nu}{r_2^2}\right) b_1 + \frac{1}{r_2^2(1-\nu)} b_2 + \left(\frac{1}{r_1^2} + \frac{1}{r_2^2}\right) b_3}$$

$$C_1^2 = \frac{1}{r_2^2(1-\nu)}\left[b_2 + \left[-(1+\nu) b_1 + b_3\right]\cancel{(1-\nu)}\right] \Rightarrow$$

$$\Rightarrow \boxed{C_1^2 = -\frac{1+\nu}{r_2^2} b_1 + \frac{1}{r_2^2(1-\nu)} b_2 + \frac{1}{r_2^2} b_3}$$

$$\sigma_{r,a} = \pm \frac{6 M_r}{h^2} \quad , \quad \sigma_{t,a} = \pm \frac{6 M_t}{h^2} \quad , \quad B = \frac{E h^3}{12(1-\nu^2)}$$
