

# Continuum mechanics

## Lecture 3

Tomáš Profant

Brno University of Technology  
Faculty of Mechanical Engineering

# First Law of Thermodynamics

The first law of thermodynamics is commonly known as the principle of balance of energy and it can be regarded as a statement of the interconvertibility of heat and work. The law does not place any restriction on the direction of the process

$$\frac{d}{dt}(K + U) = W + H,$$

where  $K$  denotes the kinetic energy,  $U$  is the internal energy,  $W$  is the power input and  $H$  is the heat input to the system.

# First Law of Thermodynamics

The kinetic energy of the system is given by

$$K = \int_{\Omega} \rho \frac{\partial \mathbf{u}}{\partial t} \cdot \frac{\partial \mathbf{u}}{\partial t} d\Omega.$$

The kinetic energy  $K$  of a system is the energy associated with the macroscopically observable velocity of the continuum. If  $\hat{U}_0$  is the energy per unit mass (or *specific internal energy*), the total internal energy of the system is given by

$$U = \int_{\Omega} \rho \hat{U}_0 d\Omega.$$

The elastic strain energy, the kinetic energy associated with the microscopic motions of molecules of the continuum and other forms of energy are also parts of internal energy.

# First Law of Thermodynamics

The power input in the nonpolar case, consists of the rate at which the external surface tractions  $\mathbf{t}$  per unit area and body force  $\mathbf{f}$  per unit volume are doing work on the mass system instantaneously occupying the volume  $\Omega$  bounded by  $S$

$$\begin{aligned}W &= \int_S \mathbf{t} \cdot \frac{\partial \mathbf{u}}{\partial t} dS + \int_{\Omega} \mathbf{f} \cdot \frac{\partial \mathbf{u}}{\partial t} d\Omega \\&= \int_S (\mathbf{n} \cdot \boldsymbol{\sigma} \cdot \frac{\partial \mathbf{u}}{\partial t}) dS + \int_{\Omega} \mathbf{f} \cdot \frac{\partial \mathbf{u}}{\partial t} d\Omega \\&= \int_{\Omega} \left[ \nabla \cdot \left( \boldsymbol{\sigma} \cdot \frac{\partial \mathbf{u}}{\partial t} \right) + \mathbf{f} \cdot \frac{\partial \mathbf{u}}{\partial t} \right] d\Omega \\&= \int_{\Omega} \left[ (\nabla \cdot \boldsymbol{\sigma} + \mathbf{f}) \cdot \frac{\partial \mathbf{u}}{\partial t} + \boldsymbol{\sigma} : \nabla \frac{\partial \mathbf{u}}{\partial t} \right] d\Omega,\end{aligned}$$

where  $:$  denotes the *double-dot* product of tensors.

# First Law of Thermodynamics

The previous expression can be written in the form

$$W = \frac{d}{dt} \int_{\Omega} \frac{\rho}{2} \frac{\partial u_i}{\partial t} \frac{\partial u_i}{\partial t} d\Omega + \int_{\Omega} \sigma_{ij} \frac{\partial e_{ij}}{\partial t} d\Omega.$$

# First Law of Thermodynamics

The rate of heat input consists of conduction through the surface  $S$  and internal heat generation in the body

$$H = - \int_S \mathbf{q} \cdot \mathbf{n} dS + \int_{\Omega} \rho Q d\Omega,$$

where  $\mathbf{q}$  is the heat flux vector normal to the surface and  $Q$  is the internal heat source per mass.

## First Law of Thermodynamics

The substitution of previous expressions for  $K$ ,  $U$ ,  $W$  and  $H$  into the energy balance

$$\frac{d}{dt}(K + U) = W + H,$$

leads to

$$\int_{\Omega} \left[ \rho d\hat{U}_0 - \sigma_{ij} de_{ij} + \left( \frac{\partial q_i}{\partial x_i} - \rho Q \right) dt \right] d\Omega = 0$$

for arbitrary volume  $\Omega$ . Let us denote

$$\rho d\hat{U}_0 = dU_0$$

Thus the conservation of energy implied by the first law of thermodynamics is given as

$$dU_0 = \sigma_{ij} de_{ij} + \left( \rho Q - \frac{\partial q_i}{\partial x_i} \right) dt.$$

## Second Law of Thermodynamics

From our experience we know, that mechanical energy that is converted into heat cannot be converted back into mechanical energy. Although the first law of thermodynamics does not restrict the reversal process, namely the conversion of heat to internal energy and internal energy to motion, such a reversal cannot occur because the frictional dissipation is an *irreversible process*. The second law of thermodynamics for a reversible process states, that there exists a function  $\eta = \eta(Q, \mathbf{q}, T)$  called the *specific entropy*, such that

$$d\eta = \left[ -\frac{1}{\rho} \nabla \cdot \left( \frac{1}{T} \mathbf{q} \right) + \frac{Q}{T} \right] dt$$

is a perfect differential. The  $d\eta$  is called the *entropy equation of state*.

## Second Law of Thermodynamics

Combining the specific entropy state

$$d\eta = \left[ -\frac{1}{\rho} \nabla \cdot \left( \frac{1}{T} \mathbf{q} \right) + \frac{Q}{T} \right] dt$$

with the first law of thermodynamics

$$dU_0 = \sigma_{ij} de_{ij} + \left( \rho Q - \frac{\partial q_i}{\partial x_i} \right) dt,$$

it is obtained

$$\sigma_{ij} de_{ij} + \rho T d\eta = dU_0.$$

## Second Law of Thermodynamics

For irreversible processes, the second law of thermodynamics states that  $d\eta$  is not total differential, but

$$\frac{\partial \eta}{\partial t} \geq -\frac{1}{\rho} \nabla \cdot \left( \frac{1}{T} \mathbf{q} \right) + \frac{Q}{T},$$

which is known as the *Clausius-Duhem inequality*.

Thank you!