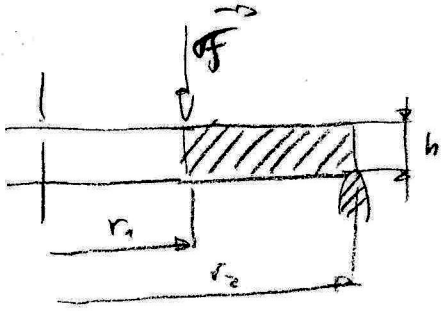
Př: Určete bezpečnost k MS pružnosti u desky podle obr. a max. průhyb



mat.: $\nu = 0.3$

$E = 2.1 \cdot 10^5 \text{ MPa}$

geom: $r_1 = 10 \text{ mm}$

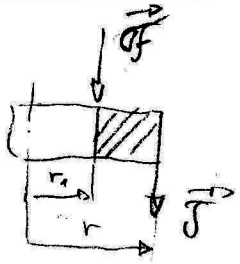
$r_2 = 20 \text{ mm}$

$h = 1 \text{ mm}$

zatížení: $q_F = \frac{1}{2\pi r_1} = \frac{1}{2\pi 10} = 1.59 \cdot 10^{-2} \text{ N} \cdot \text{mm}^{-1}$

tuhost desky: $B = \frac{Eh^3}{12(1-\nu^2)} = \frac{2.1 \cdot 10^5 \cdot 1^3}{12(1-0.3^2)} = 1.92 \cdot 10^4 \text{ N} \cdot \text{mm}$

1. Uvolnění elementu



$r 2\pi T = -2\pi r_1 q_F = -1$

$T = -\frac{1}{2\pi r} = -1.59 \cdot 10^{-1} \frac{1}{r}$

2. Partikulární řešení

$$\vartheta_p = -\frac{1}{r \cdot B} \int r \int T \, dr\,dr = -\frac{1}{1.92 \cdot 10^4 \cdot r} \int r \cdot \int \left(-1.59 \cdot 10^{-1} \cdot \frac{1}{r}\right) dr\,dr =$$

$$= 8.28^{-6} \frac{1}{r} \int r \ln r \, dr = 8.28^{-6} \cdot \frac{1}{r} \left[0.5 \cdot r^2 (\ln r - 0.5)\right] = 4.14 \cdot 10^{-6} r (\ln r - 0.5)$$

3. Natočení, momenty a průhyb

$$\vartheta = C_1 r + \frac{C_2}{r} + 4.14 \cdot 10^{-6} r (\ln r - 0.5)$$

$$\frac{d\vartheta}{dr} = C_1 + \left(-\frac{C_2}{r^2}\right) + 4.14 \cdot 10^{-6} (\ln r - 0.5) + 4.14 \cdot 10^{-6} r \cdot \frac{1}{r} = C_1 - \frac{C_2}{r^2} + 4.14 \cdot 10^{-6} (\ln r + 0.5)$$

$$\frac{\vartheta}{r} = C_1 + \frac{C_2}{r^2} + 4.14 \cdot 10^{-6} (\ln r - 0.5)$$

$$M_r = -B \left(\frac{d\vartheta}{dr} + \nu \frac{\vartheta}{r}\right) = -1.92 \cdot 10^4 \left(C_1 - \frac{C_2}{r^2} + 4.14 \cdot 10^{-6} (\ln r + 0.5) + 0.3 \left[C_1 + \frac{C_2}{r^2} + \right.\right.$$

$$\left.\left. + 4.14 \cdot 10^{-6} (\ln r - 0.5)\right]\right) = -1.92 \cdot 10^4 \left(1.3 C_1 - 0.4 \cdot \frac{C_2}{r^2} + 5.38 \cdot 10^{-6} \ln r + 1.45 \cdot 10^{-6}\right)$$

$$w = \int \vartheta \, dr + C_3 = C_1 0.5 \cdot r^2 + C_2 \ln r + 4.14 \cdot 10^{-6} \cdot \left(0.5 \cdot r^2 (\ln r - 0.5) - 0.25 r^2\right) +$$

$$+ C_3 = 0.5 r^2 \cdot C_1 + \ln r \cdot C_2 + 2.07 \cdot 10^{-6} r^2 \ln r - 2.07 \cdot 10^{-6} r^2 + C_3$$

4. Okr. podmínky

$$M_r\big|_{r=r_1} = 0 \;,\; M_r\big|_{r=r_2} = 0 \;,\; w\big|_{r=r_2} = 0$$

5. Stanovení C_1, C_2, C_3

$$0 = 1.3C_1 - 0.7 \cdot \frac{C_2}{10^2} + 5.38 \cdot 10^{-6} \ln 10 + 1.45 \cdot 10^{-6}$$

$$0 = 1.3C_1 - 0.4 \cdot \frac{C_2}{20^2} + 5.38 \cdot 10^{-6} \ln 20 + 1.45 \cdot 10^{-6}$$

$$0 = 0.7 \cdot 20^2 C_1 + \ln 20 \cdot C_2 + 2.07 \cdot 10^{-6} 20^2 \ln 20 - 2.07 \cdot 10^{-6} \cdot 20^2 + C_3$$

$$1.3C_1 - 4 \cdot 10^{-3} C_2 = -1.38 \cdot 10^{-5}$$

$$1.3C_1 - 1.45 \cdot 10^{-3} C_2 = -1.46 \cdot 10^{-5}$$

$$C_3 = -200 C_1 - 3C_2 - 1.65 \cdot 10^{-3}$$

$$C_1 = \frac{4 \cdot 10^{-3} C_2 - 1.38 \cdot 10^{-5}}{1.3}$$

$$1.3 \cdot \frac{4 \cdot 10^{-3} C_2 - 1.38 \cdot 10^{-5}}{1.3} - 1.45 \cdot 10^{-3} C_2 = -1.46 \cdot 10^{-5}$$

$$5.25 \cdot 10^{-3} C_2 = -3.8 \cdot 10^{-6}$$

$$C_2 = -7.24 \cdot 10^{-4}$$

$$C_1 = 5.38 \cdot 10^{-3} \cdot (-7.24 \cdot 10^{-4}) - 1.06 \cdot 10^{-5} = -1.45 \cdot 10^{-5}$$

$$C_3 = -200 \cdot (-1.45 \cdot 10^{-5}) - 3 \cdot (-7.24 \cdot 10^{-4}) - 1.65 \cdot 10^{-3} = 3.41 \cdot 10^{-3}$$

6. Řešení

$$\sigma_{r,ec} = \frac{C M_r}{h^2} = 6 \cdot (-1.92 \cdot 10^{-4}) \cdot \left(1.3 \cdot (-1.45 \cdot 10^{-5}) - 0.7 \cdot (-7.24 \cdot 10^{-4}) \cdot \frac{1}{r^2} + \right.$$

$$\left. + 5.38 \cdot 10^{-6} \ln r + 1.45 \cdot 10^{-6} \right) = -1.15 \cdot 10^{-5} \left(-1.74 \cdot 10^{-5} + 5.07 \cdot 10^{-4} \frac{1}{r^2} + 5.38 \cdot 10^{-6} \ln r \right.$$

$$M_t = -B \left(\frac{v}{r} + v \cdot \frac{dv}{dr} \right) = -1.92 \cdot 10^{+4} \left((1+v) \cdot C_1 + (1-v) \frac{C_2}{r^2} + (1+v) 4.14 \cdot 10^{-6} \ln r \right.$$

$$\left. - (1-v) 0.5 \cdot 4.14 \cdot 10^{-6} \right) = -1.92 \cdot 10^{+4} \left(1.3 \cdot (-1.45 \cdot 10^{-5}) + 0.7 (-7.24 \cdot 10^{-4}) \cdot \frac{1}{r^2} \right.$$

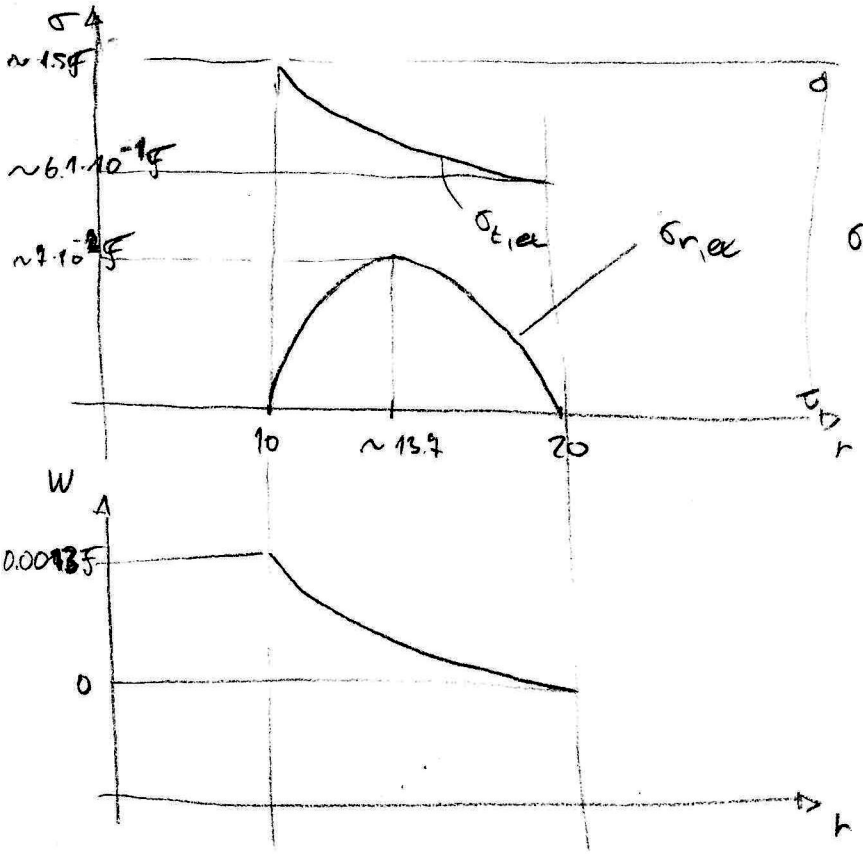
$$\left. + 1.3 \cdot 4.14 \cdot 10^{-6} \ln r - 0.7 \cdot 0.5 \cdot 4.14 \cdot 10^{-6} \right) = -1.92 \cdot 10^{+4} \cdot \left(-2.03 \cdot 10^{-5} - 5.07 \cdot 10^{-4} \frac{1}{r^2} \right.$$

$$\left. + 5.38 \cdot 10^{-6} \ln r \right)$$

$$\sigma_{t,ec} = \frac{C M_t}{h^2} = -1.15 \cdot 10^{+5} \left(-2.03 \cdot 10^{-5} - 5.07 \cdot 10^{-4} \frac{1}{r^2} + 5.38 \cdot 10^{-6} \ln r \right)$$

$$w = 0.5 \cdot (-1.45 \cdot 10^{-5}) r^2 + (-7.24 \cdot 10^{-4}) \ln r + 2.07 \cdot 10^{-6} r^2 \ln r - 2.07 \cdot 10^{-6} r^2 + 3.41 \cdot 10^{-3}$$

$$= -9.32 \cdot 10^{-6} \cdot r^2 - 7.24 \cdot 10^{-4} \ln r + 2.07 \cdot 10^{-6} r^2 \ln r + 3.41 \cdot 10^{-3}$$



$\sigma_{ekv} = |\sigma_{r,ek} - \sigma_{t,ek}| = |\sigma_{t,ek}| = 1.5F$ MPa

$k = \frac{\sigma_k}{\sigma_{ekv}}$

$W_{max} = 0.0013 \text{ mm} \cdot F$