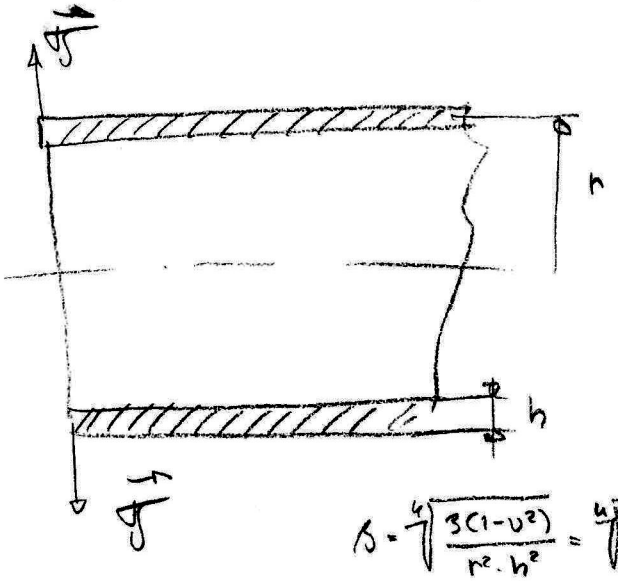
Př: k MS pružnosti určete bezpečnost a uvažujte válcové skořepiny



mat.: $\nu = 0.3$
$E = 2.1 \cdot 10^5$ MPa

geom.: $r = 20$ mm
$h = 1$ mm

zatížení: $q \cdot 2\pi r = 1 \Rightarrow q = \frac{1}{2\pi \cdot 20} = 4.96 \cdot 10^{-3} \frac{N}{mm}$

$p = p_z = 0$

$\beta = \sqrt[4]{\frac{3(1-\nu^2)}{r^2 \cdot h^2}} = \sqrt[4]{\frac{3(1-0.3^2)}{20^2 \cdot 1^2}} = 2.84 \cdot 10^{-1} \frac{1}{mm}$, $B = \frac{Eh^3}{12(1-\nu^2)} = \frac{2.1 \cdot 10^5 \cdot 1^3}{12(1-0.3^2)} = 1.92 \cdot 10^4 \cdot N mm$

1. Partikulární řešení

$$u_p = \frac{r^2}{E}\left[p - \frac{\nu}{r}\left(c_0 - \int p_z dz\right)\right] = -\frac{r\nu}{E} \cdot c_0 = \frac{20 \cdot 0.3}{2.1 \cdot 10^5} c_0 = 2.86 \cdot 10^{-5} c_0$$

2. Natočení, momenty...

$$u = u_p + \tilde{u} = e^{-\beta z}(c_1 \sin \beta z + c_2 \cos \beta z) + 2.86 \cdot 10^{-5} c_0 = e^{-2.84 \cdot 10^{-1} z}(c_1 \sin 2.84 \cdot 10^{-1} z + c_2 \cos 2.84 \cdot 10^{-1} z) + 2.86 \cdot 10^{-5} c_0$$

$$M_z = +2B\beta^2 e^{-\beta z}(-c_1 \cos \beta z + c_2 \sin \beta z) - B \frac{d^2}{dz^2} u_p = -2 \cdot 1.92 \cdot 10^4 \cdot (2.84 \cdot 10^{-1})^2 e^{-2.84 \cdot 10^{-1} z} \cdot (-c_1 \cos 2.84 \cdot 10^{-1} z + c_2 \sin 2.84 \cdot 10^{-1} z) - 0 = -3.16 \cdot 10^3 e^{-2.84 \cdot 10^{-1} z} \cdot (-c_1 \cdot \cos 2.84 \cdot 10^{-1} z + c_2 \sin 2.84 \cdot 10^{-1} z)$$

$$q_{rz} = -2B\beta^3 e^{-\beta z}[c_1(\sin \beta z + \cos \beta z) + c_2(-\sin \beta z + \cos \beta z)] - B \cdot \frac{d^3}{dz^3} u_p = -2 \cdot 1.92 \cdot 10^4 \cdot (2.84 \cdot 10^{-1})^3 \cdot e^{-2.84 \cdot 10^{-1} z}[c_1(\sin 2.84 \cdot 10^{-1} z + \cos 2.84 \cdot 10^{-1} z) + c_2(-\sin 2.84 \cdot 10^{-1} z + \cos 2.84 \cdot 10^{-1} z)] = -9.08 \cdot 10^2 e^{-2.84 \cdot 10^{-1} z}[c_1(\sin 2.84 \cdot 10^{-1} z + \cos 2.84 \cdot 10^{-1} z) + c_2(-\sin 2.84 \cdot 10^{-1} z + \cos 2.84 \cdot 10^{-1} z)]$$

$$N_z = c_0 - \int p_z dz = c_0$$

3. Okr. podmínky

$$M_z|_{z=0} = 0, \quad q_{rz}|_{z=0} = -q, \quad N_z|_{z=0} = 0$$

4. Nalezení c_0, c_1, c_2:

$$-3.16 \cdot 10^3 (-c_1 + 0) = 0 \Rightarrow c_1 = 0$$

$$-9.08 \cdot 10^2 (c_1 + c_2) = -4.96 \cdot 10^{-3} \Rightarrow c_2 = +\frac{4.96 \cdot 10^{-3}}{9.08 \cdot 10^2} = +8.47 \cdot 10^{-6}$$

$$c_0 = 0$$

5. Řešení

$$u = e^{-2.87\cdot10^{-1}z}\cdot(+8.77\cdot10^{-6})\cdot\cos(2.87\cdot10^{-1}z) = +8.77\cdot10^{-6}e^{-2.87\cdot10^{-1}z}\cos 2.87\cdot10^{-1}z$$

$M_z = 0$

$$w = -\frac{v}{r}\int u\,dz = -\frac{v}{r}\left\{\left(-\frac{1}{2\beta}\right)e^{-\beta z}\left[(c_1+c_2)\cos\beta z + (c_1-c_2)\sin\beta z\right] + 2.86\cdot10^{-5}\cos\right\}$$

$$= -\frac{0.3}{20}\left\{\left(-\frac{1}{2\cdot2.87\cdot10^{-1}}\right)e^{-2.87\cdot10^{-1}z}\left[+8.44\cdot10^{-6}\cos 2.87\cdot10^{-1}z + 8.77\cdot10^{-6}\sin 2.87\cdot10^{-1}z\right]\right\}$$

$$= 2.29\cdot10^{-7}\,e^{-2.87\cdot10^{-1}z}\left[+\cos 2.87\cdot10^{-1}z + \sin 2.87\cdot10^{-1}z\right]$$

$$\sigma_{rz} = -2\cdot1.92\cdot10^{3}\cdot(2.87\cdot10^{-1})^{2}\,e^{-2.87\cdot10^{-1}z}\left[+8.44\cdot10^{-6}\left(-\sin 2.87\cdot10^{-1}z + \cos 2.87\cdot10^{-1}z\right)\right] =$$

$$= -4.96\cdot10^{-3}\,e^{-2.87\cdot10^{-1}z}\left(\cos 2.87\cdot10^{-1}z - \sin 2.87\cdot10^{-1}z\right)$$

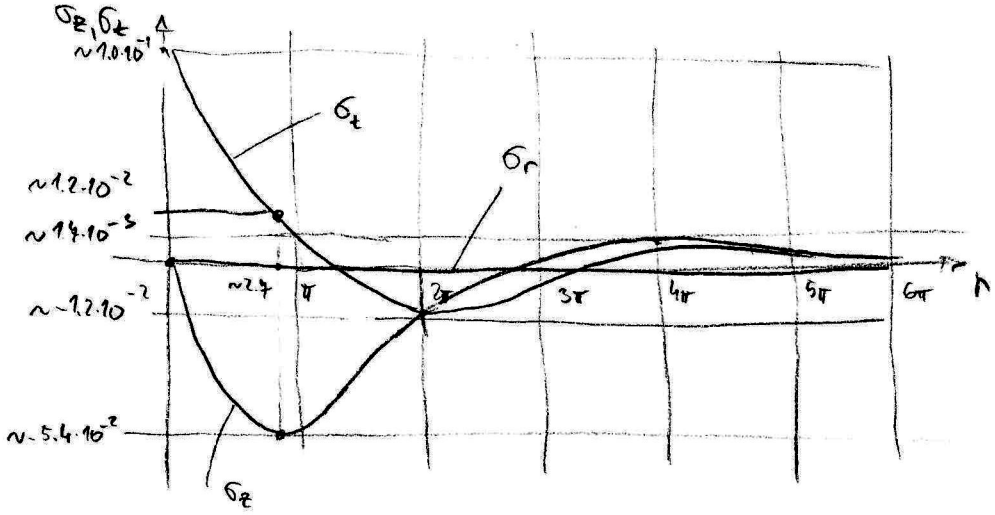
$$N_t = r\left(p_n + \frac{d\sigma_{rz}}{dz}\right) = 20\cdot\left(-4.96\cdot10^{-3}\cdot(-2.87\cdot10^{-1})\,e^{-2.87\cdot10^{-1}z}\left(\cos 2.87\cdot10^{-1}z - \sin 2.87\cdot10^{-1}z\right)\right.$$
$$\left. - 4.96\cdot10^{-3}e^{-2.87\cdot10^{-1}z}\left(-2.87\cdot10^{-1}\cdot\sin 2.87\cdot10^{-1}z - 2.87\cdot10^{-1}\cos 2.87\cdot10^{-1}z\right)\right) = 4.57\cdot10^{-2}$$
$$\cdot\, e^{-2.87\cdot10^{-1}z}\left(2\cdot\cos 2.87\cdot10^{-1}z\right) = 9.14\cdot10^{-2}e^{-2.87\cdot10^{-1}z}\cos 2.87\cdot10^{-1}z$$

$$M_z = -3.16\cdot10^{-3}\cdot e^{-2.87\cdot10^{-1}z}\cdot 8.77\cdot10^{-6}\sin 2.87\cdot10^{-1}z = -2.77\cdot10^{-2}e^{-2.87\cdot10^{-1}z}\sin 2.87\cdot10^{-1}z$$

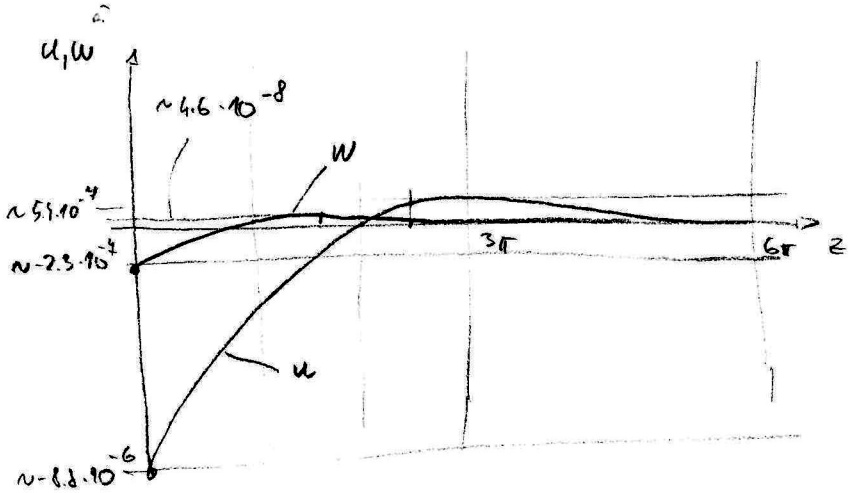
$$M_t = 0.3\cdot M_z = -8.31\cdot10^{-3}\,e^{-2.87\cdot10^{-1}z}\sin 2.87\cdot10^{-1}z$$

$$\sigma_{z,ea} = \frac{M_z}{h} \pm \frac{6M_z}{h^2} = \pm\frac{6}{1^2}\cdot(-2.77\cdot10^{-2}e^{-2.87\cdot10^{-1}z}\sin 2.87\cdot10^{-1}z) = \mp 1.66\cdot10^{-1}e^{-2.87\cdot10^{-1}z}\cdot$$
$$\cdot\sin 2.87\cdot10^{-1}z$$

$$\sigma_{t,ea} = \frac{M_t}{h} \pm \frac{6M_t}{h^2} = 9.14\cdot10^{-2}e^{-2.87\cdot10^{-1}z}\cos 2.87\cdot10^{-1}z \pm 6\cdot(-8.31\cdot10^{-3}e^{-2.87\cdot10^{-1}z}\sin 2.87\cdot10^{-1}z$$
$$= e^{-2.87\cdot10^{-1}z}\left[9.14\cdot10^{-2}\cos 2.87\cdot10^{-1}z \mp 4.99\cdot10^{-2}\sin 2.87\cdot10^{-1}z\right]$$



$$\sigma_{red} = \max\left\{|\sigma_z-\sigma_t|, |\sigma_z-\sigma_r|, |\sigma_t-\sigma_r|\right\} = |\sigma_t|\Big|_{z=0} \doteq 1.0\cdot10^{-1}\ \text{MPa}$$



$$\underline{u}_{max} \approx \sqrt{(8.8 \cdot 10^{-6})^2 + (2.3 \cdot 10^{-4})^2}$$

$$\approx 8.8 \cdot 10^{-6} \text{ mm}$$