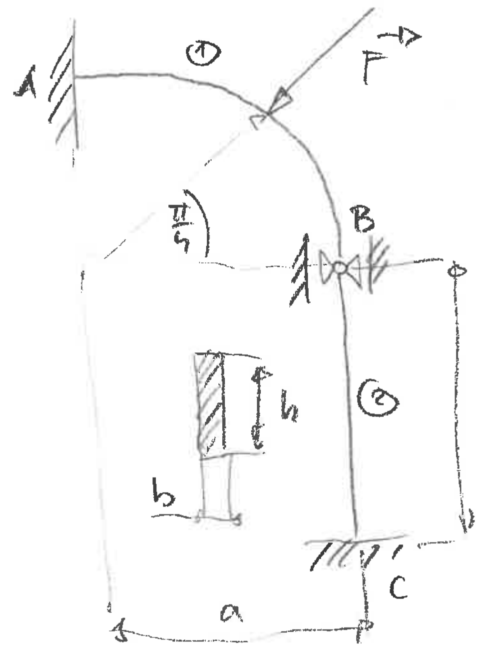
Př: Určete bezpečnost vzhledem k MS pružnosti a vzpěrné stability.



$b = 10\,mm$

$h = 100\,mm$

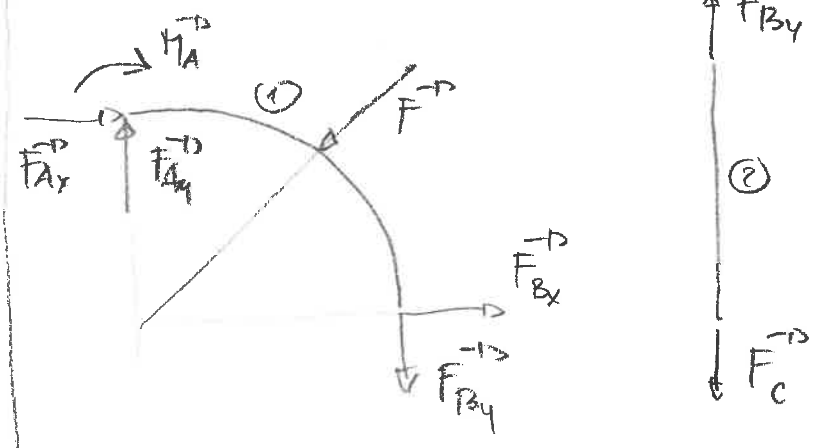
$a = 3000\,mm$

$F = 1000\,N$

$E = 2{,}1 \cdot 10^5\,MPa$

$\sigma_k = 500\,MPa$

1. Úplné uvolnění

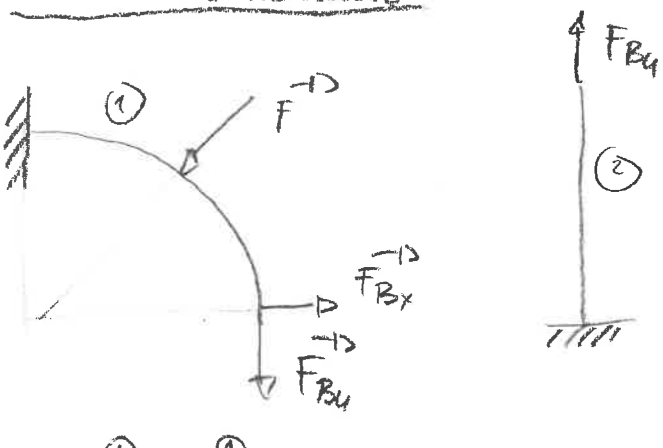


2. Statický rozbor

$NP = \{F_{Ax}, F_{Ay}, M_A, F_{Bx}, F_{By}, F_C\}$

$$\left.\begin{matrix} \mu = 6 \\ \nu = 3+1 \end{matrix}\right\} \Rightarrow s = \mu - \nu = 2$$

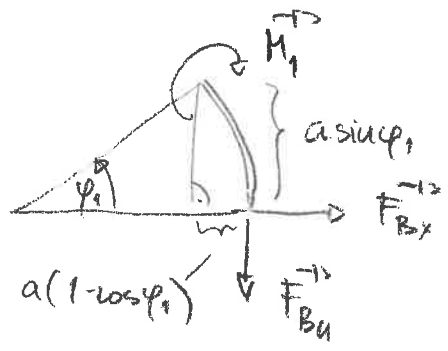
3. Částečné uvolnění



$$u_B^{①} + u_B^{①} = 0$$

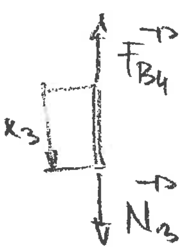
$$w_B^{①} = 0$$

4. VVÚ



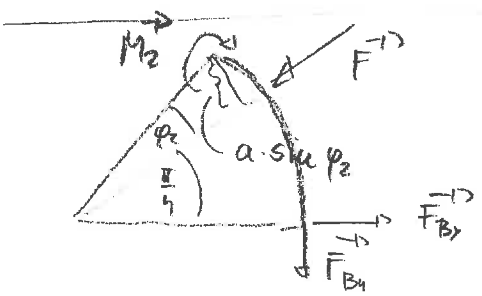
$\varphi_1 \in \langle 0, \frac{\pi}{4})$

$M_1 = -F_{By} \cdot a(1-\cos\varphi_1) + F_{Bx} \cdot a \sin\varphi_1$



$x_3 \in \langle 0, a)$

$N_3 = F_{By}$



$\varphi_2 \in \langle 0, \frac{\pi}{4})$

$$M_2 = -F_{By} a\left(1-\cos\left(\varphi_2+\frac{\pi}{4}\right)\right) + F_{Bx} a \sin\left(\varphi_2+\frac{\pi}{4}\right) - F \cdot a \sin\varphi_2$$

5. Odstranění stat. neurč.

$$\int_0^{\frac{\pi}{4}} \frac{M_1}{\cancel{E}J}\frac{\partial M_1}{F_{By}}\cancel{a}\,d\varphi_1 + \int_0^{\frac{\pi}{4}} \frac{M_2}{\cancel{E}J}\frac{\partial M_2}{\partial F_{By}}\cancel{a}\,d\varphi_2 + \frac{N_3 a}{\cancel{E}S}\frac{\partial N_3}{\partial F_{By}} = 0$$

$$\int_0^{\frac{\pi}{2}} \frac{M_1}{\cancel{E}J}\frac{\partial M_2}{\partial F_{Bx}}\cancel{a}\,d\varphi_1 + \int_0^{\frac{\pi}{4}} \frac{M_2}{\cancel{E}J}\frac{\partial M_2}{\partial F_{Bx}}\cancel{a}\,d\varphi_2 = 0$$

$$S\int_0^{\frac{\pi}{4}} \left(-F_{By}\cdot a(1-\cos\varphi_1) + F_{Bx}a\sin\varphi_1\right)\cdot(-a)(1-\cos\varphi_1)\,d\varphi_1 +$$

$$+S\int_0^{\frac{\pi}{4}} \left(-F_{By}a\left[1-\cos\left(\varphi_2+\frac{\pi}{4}\right)\right] + F_{Bx}a\sin\left(\varphi_2+\frac{\pi}{4}\right)\right)(-a)\left[1-\cos\left(\varphi_2+\frac{\pi}{4}\right)\right]d\varphi_2 +$$

$$+\ J\cdot F_{By}\cdot 1 = 0$$

$-F\cdot a\sin\varphi_2$

$$\int_0^{\frac{\pi}{4}} \left(-F_{By}a(1-\cos\varphi_1) + F_{Bx}a\sin\varphi_1\right)\cdot a\cdot\sin\varphi_1\,d\varphi_1 + \int_0^{\frac{\pi}{4}}\left(-F_{By}a\left[1-\cos\left(\varphi_2+\frac{\pi}{4}\right)\right]+\right.$$

$$\left.+F_{Bx}a\sin\left(\varphi_2+\frac{\pi}{4}\right) - F\cdot a\sin\varphi_2\right)\cdot a\cdot\sin\left(\varphi_2+\frac{\pi}{4}\right)d\varphi_2 = 0$$

Použité integrály:

$$\int(1-\cos\varphi_1)^2 d\varphi_1 = \int\left(1-2\cos\varphi_1+\cos^2\varphi_1\right)d\varphi_1 = \varphi_1 - 2\sin\varphi_1 + \frac{1}{2}\left(\varphi_1+\frac{1}{2}\sin 2\varphi_1\right) + C$$

$$\int\sin\varphi_1(1-\cos\varphi_1)\,d\varphi_1 = \int\left(\sin\varphi_1 - \sin\varphi_1\cos\varphi_1\right)d\varphi_1 = -\cos\varphi_1 + \frac{1}{4}\cos 2\varphi_1 + C$$

$$\int\sin\varphi_2\left(1-\cos\left(\varphi_2+\frac{\pi}{4}\right)\right)d\varphi_2 = \int\left[\sin\varphi_2 - \sin\varphi_2\left(\cos\varphi_2\cdot\cos\frac{\pi}{4} - \sin\varphi_2\sin\frac{\pi}{4}\right)\right]d\varphi_2 =$$

$$= \int\left[\sin\varphi_2 - \sin\varphi_2\cos\varphi_2\cdot\frac{\sqrt{2}}{2} + \sin^2\varphi_2\frac{\sqrt{2}}{2}\right]d\varphi_2 = -\cos\varphi_2 + \sqrt{2}\,\frac{1}{2}\cos 2\varphi_2 + \frac{1}{2}\left(\varphi_2 -\right.$$

$$\left.-\frac{1}{2}\sin 2\varphi_2\right)\cdot\frac{\sqrt{2}}{2} + C$$

$$\int\sin^2\varphi_1\,d\varphi_1 = \frac{1}{2}\left(\varphi_1 - \frac{1}{2}\sin 2\varphi_1\right) + C$$

$$\int\sin\varphi_2\sin\left(\varphi_2+\frac{\pi}{4}\right)d\varphi_2 = \int\sin\varphi_2\left(\sin\varphi_2\cos\frac{\pi}{4} + \cos\varphi_2\cdot\sin\frac{\pi}{4}\right)d\varphi_2 = \int\left[\frac{\sqrt{2}}{2}\sin^2\varphi_2 +\right.$$

$$\left.+\frac{\sqrt{2}}{2}\sin\varphi_2\cos\varphi_2\right]d\varphi_2 = \frac{\sqrt{2}}{2}\cdot\frac{1}{2}\left(\varphi_2 - \frac{1}{2}\sin 2\varphi_2\right) - \frac{\sqrt{2}}{2}\cdot\frac{1}{4}\cos 2\varphi_2 + C$$

$$S\left[F_{By}a^2\left(\varphi_1 - 2\sin\varphi_1 + \frac{1}{2}\left(\varphi_1+\frac{1}{2}\sin 2\varphi_1\right)\right) - F_{Bx}a^2\left(-\cos\varphi_1 + \frac{1}{4}\cos 2\varphi_1\right)\right]_0^{\frac{\pi}{4}}$$

$$+S\left[F_{By}a^2\left(\varphi_2+\frac{\pi}{4} - 2\sin\left(\varphi_2+\frac{\pi}{4}\right) + \frac{1}{2}\left(\varphi_2+\frac{\pi}{4}+\frac{1}{2}\sin 2\left(\varphi_2+\frac{\pi}{4}\right)\right)\right) - F_{Bx}a^2\left(-\cos\left(\varphi_2+\frac{\pi}{4}\right)+\right.\right.$$

$$\left.\left.+\frac{1}{4}\cos 2\left(\varphi_2+\frac{\pi}{4}\right)\right) + F\cdot a^2\left(-\cos\varphi_2 + \frac{\sqrt{2}}{2}\cos 2\varphi_2 + \frac{1}{2}\left(\varphi_2 - \frac{1}{2}\sin 2\varphi_2\right)\cdot\frac{\sqrt{2}}{2}\right)\right]_0^{\frac{\pi}{4}} + J\cdot F_{By} = 0$$

$$\left[-F_{By}\cancel{a^2}\left(-\cos\varphi_1+\frac14\cos2\varphi_1\right)+F_{Bx}\cancel{a^2}\cdot\frac12\left(\varphi_1-\frac12\sin2\varphi_1\right)\right]_0^{\frac{\pi}{4}}+\Big[-F_{By}\cancel{a^2}\cdot$$

$$\cdot\left(-\cos\left(\varphi_2+\frac{\pi}{4}\right)+\frac14\cos2\left(\varphi_2+\frac{\pi}{4}\right)\right)+F_{Bx}\cancel{a^2}\frac12\left(\varphi_2+\frac{\pi}{4}-\frac12\sin2\left(\varphi_2+\frac{\pi}{4}\right)\right)-F\cancel{a^2}\cdot$$

$$\cdot\left(\frac{\sqrt2}{4}\left(\varphi_2-\frac12\sin2\varphi_2\right)-\frac{\sqrt2}{8}\cos2\varphi_2\right)\Big]_0^{\frac{\pi}{4}}=0$$

$$-2+\cancel{\sqrt2}\qquad \frac{\pi}{8}+\frac{\pi}{8}+\frac12-\frac{\sqrt2}{4}$$

$$S\cdot F_{By}\cancel{a^2}\left(\frac{\pi}{4}-\cancel{2}\cdot\frac{\sqrt2}{\cancel{2}}+\frac12\cdot\frac{\pi}{4}+\frac14\cdot1+\frac{\pi}{4}+\frac{\pi}{4}-2\left(1-\frac{\sqrt2}{2}\right)+\frac12\left(\frac{\pi}{4}+\frac{\pi}{4}+\frac12\left(1-\frac{\sqrt2}{2}\right)\right)+\right.$$

$$\left.+\frac{J}{a^2}\right)+SF_{Bx}\cancel{a^2}\left(\cancel{\frac{\sqrt2}{2}}-1-\frac14(-1)-\cancel{\frac{\sqrt2}{2}}-\frac14(-1)\right)+SF\cancel{a^2}\left(-\cancel{\frac{\sqrt2}{2}}+1+\cancel{\frac{\sqrt2}{2}}\cdot1+\frac{\sqrt2}{4}\cdot\right.$$

$$\frac{\sqrt2\pi}{16}-\frac{\sqrt2}{8}$$

$$\left.\cdot\left(\frac{\pi}{4}-\frac12\cdot1\right)\right)=0$$

$$F_{By}\left(\cancel{\frac{\sqrt2}{2}}-1-\frac14(-1)-\cancel{\frac{\sqrt2}{2}}-\frac14(-1)\right)+F_{Bx}\left(\frac12\left(\frac{\pi}{2}-\frac12\cdot(-1)\right)\right)-F\cdot\left(\frac{\sqrt2}{4}\left(\frac{\pi}{4}-\right.\right.$$

$$\frac{\sqrt2\pi}{16}+\frac{\sqrt2}{8}$$

$$\left.\left.-\frac12\cdot1\right)-\frac{\sqrt2}{8}(-1)\right)=0$$

$$SF_{By}\left(-\frac54+\frac{11\pi}{8}-\frac{\sqrt2}{4}+\frac{J}{a^2}\right)+SF_{Bx}\left(-\frac12\right)+F\left(1+\frac{\sqrt2\pi}{16}-\frac{\sqrt2}{8}\right)=0$$

$$F_{By}\left(-\frac12\right)+F_{Bx}\left(\frac14+\frac{\pi}{4}\right)-F\left(\frac{\sqrt2}{4}+\frac{\sqrt2\pi}{16}\right)=0$$

$S=b\cdot h=1000\ mm^2$

$J=\frac{1}{12}b^3h=\frac{1}{12}10^3\,100\ =8.33\cdot10^3\ mm^4$

$F_{By}\ 1.86\cdot10^3-F_{Bx}\cdot500+F\cdot1.1\ 10^3=0$

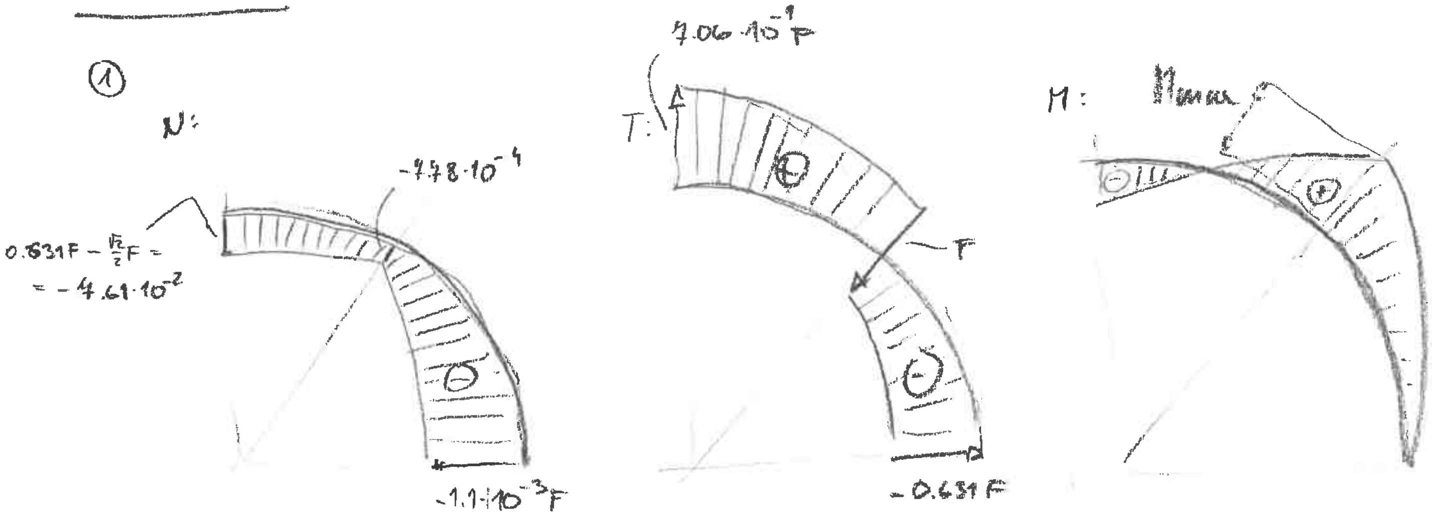
$-F_{By}\cdot0.5+F_{Bx}\cdot1.04-F\ 6.31\cdot10^{-1}=0$

$$\begin{bmatrix}2.42\cdot10^3 & -0.5\cdot10^3\\ -0.5 & 1.04\end{bmatrix}\begin{bmatrix}F_{By}\\ F_{Bx}\end{bmatrix}=\begin{bmatrix}-1.1\cdot10^3F\\ 0.631F\end{bmatrix}$$

$$\begin{bmatrix}F_{By}\\ F_{Bx}\end{bmatrix}=\begin{bmatrix}-1.1\cdot10^{-3}F\\ 0.631\cdot F\end{bmatrix}$$

6. Řešení

①



$$M_{max} = M_1\left(\varphi_1 = \frac{\pi}{4}\right) = -1.1\cdot10^{-3}F\cdot3000\left(1-\frac{\sqrt{2}}{2}\right) + 0.631F\cdot3000\cdot\frac{\sqrt{2}}{2} = 1.34F\ \text{Nmm}$$

$$k_1 = \frac{\sigma_k}{\sigma_{max}} = \frac{500}{\dfrac{1.34\cdot F}{\frac{1}{6}\cdot10^2\,100}} = \frac{500\cdot10^2\,100}{6\cdot1.34\cdot F} = 6.22\cdot10^5\,\frac{1}{F}$$

②

$|\vec{F}_{By}| = 1.1\cdot10^{-3}F$

$\alpha = \sqrt{2}\pi$

$$\lambda_k = \sqrt{\frac{\alpha^2E}{\sigma_k}} = \sqrt{\frac{2\pi^2\cdot2.1\cdot10^5}{500}} = 91.1$$

$$\lambda = \frac{L}{i} = \frac{a}{\sqrt{\frac{J}{S}}} = \frac{3000}{\sqrt{\dfrac{8.33\cdot10^3}{10\cdot100}}} = 1.04\cdot10^3$$

$\lambda > \lambda_k$ => kontrola k MS vzpěrné stability

$$F_{kr} = \frac{\alpha^2EJ}{L^2} = \frac{2\pi^2\cdot2.1\cdot10^5\cdot8.33\cdot10^3}{3000^2} = 3.84\cdot10^3\ \text{N}$$

$|F_{By}| < F_{kr}$ => nedochází k borcení střednice prutu