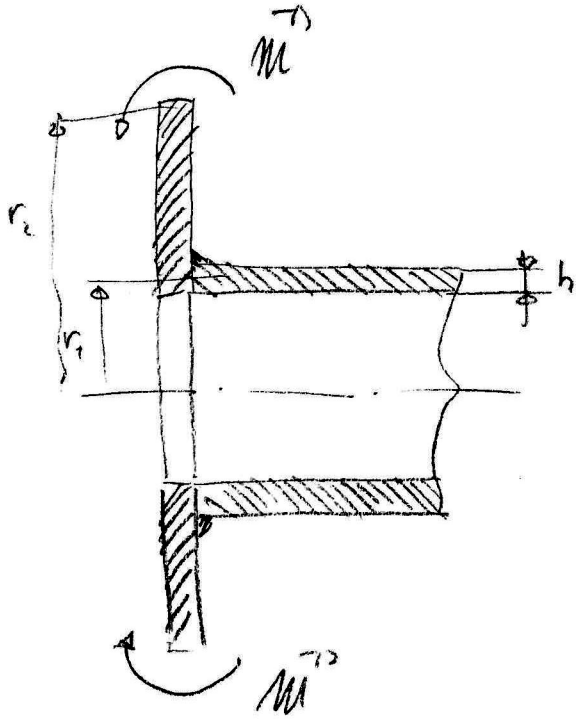
Př.: Určete bezpečnost k MS pružnosti u soustavy těles.

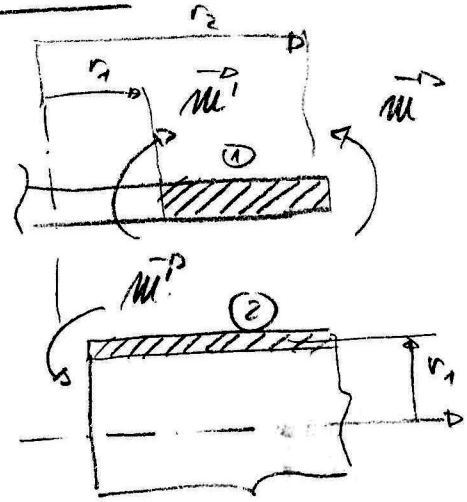


mat.: $\nu = 0.3$
$E = 2.1 \cdot 10^5 \text{ MPa}$

geom: $r_1 = 20 \text{ mm}$
$r_2 = 40 \text{ mm}$
$h = 1 \text{ mm}$

zatížení: $M \cdot 2\pi r_2 \cdot 1 \Rightarrow M = \frac{1}{2\pi \cdot 40} = 3.98 \cdot 10^{-3}$

1. Uvolnění



$w_1|_{r=r_1} = w_2|_{z=0}$

$-\vartheta_1|_{r=r_1} = \vartheta_2|_{z=0}$

u_2 – zanedbání vlivu (musel by se počítat jako stěna-deska)

2. Okr. podmínky

$M_{r_1}|_{r=r_2} = M \qquad M_{z_2}|_{z=0} = M'$

$M_{r_1}|_{r=r_1} = M' \qquad M_{z_2}|_{z=0} = 0 \quad , \quad \nabla_{r_2}|_{z=0} = 0$

3. Partikulární řešení

$\vartheta_{p_1} = 0$

$u_{p_2} = -\frac{r\nu}{E} C_0 = \frac{20 \cdot 0.3}{2.1 \cdot 10^5} C_0 = 2.86 \cdot 10^{-5} C_0$

4. Natočení, momenty, průhyb

$\vartheta_1 = C_{11} \cdot r + \frac{C_{12}}{r}$

$B_1 = \frac{E \cdot h^3}{12(1-\nu^2)} = \frac{2.1 \cdot 10^5 \cdot 1^3}{12(1-0.3^2)} = 1.92 \cdot 10^4 \frac{N}{mm} = B_2$

$\frac{d\vartheta_1}{dr} = C_{11} - \frac{C_{12}}{r^2}$

$k_2 = \sqrt[4]{\frac{3(1-\nu^2)}{r_1^2 h^2}} = \sqrt[4]{\frac{3(1-0.3^2)}{20^2 \cdot 1^2}} = 2.84 \cdot 10^{-1} \frac{1}{mm}$

$\frac{\vartheta_1}{r} = C_{11} + \frac{C_{12}}{r^2}$

$M_{r_1} = -B_1 \left(\frac{d\vartheta_1}{dr} + \nu \frac{\vartheta_1}{r} \right) = -1.92 \cdot 10^4 \left(C_{11} - \frac{C_{12}}{r^2} + 0.3\, C_{11} + 0.3\, \frac{C_{12}}{r^2} \right) =$

$= -1.92 \cdot 10^4 \left(1.3\, C_{11} - 0.7 \cdot \frac{C_{12}}{r^2} \right)$

$$w_1 = \int \vartheta_1 \, dr + C_3 = 0.5 \cdot C_{11} r^2 + C_{12} \ln r + C_3$$

$$u_2 = e^{-\beta_2 z}\left(C_{21} \cdot \sin\beta_2 z + C_{22} \cos\beta_2 z \right) + 2.86 \cdot 10^{-5} C_0 = e^{-2.84 \cdot 10^{-1} z} \cdot (C_{21} \sin 2.84 \cdot 10^{-1} z +$$

$$+ C_{22} \cos 2.84 \cdot 10^{-1} z) + 2.86 \cdot 10^{-5} C_0$$

$$\vartheta_2 = -\beta e^{-\beta z}\left[C_{21}(\sin\beta z - \cos\beta z) + C_{22}(\sin\beta z + \cos\beta z)\right] = -2.84 \cdot 10^{-1} e^{-2.84 \cdot 10^{-1} z} \cdot \left[C_{21} \cdot (\right.$$

$$\left. \sin 2.84 \cdot 10^{-1} z - \cos 2.84 \cdot 10^{-1} z) + C_{22}(\sin 2.84 \cdot 10^{-1} z + \cos 2.84 \cdot 10^{-1} z)\right]$$

$$M_{z_2} = -2\beta\beta^2 e^{-\beta z}(-C_{21}\cos\beta z + C_{22}\sin\beta z) = -2 \cdot 1.92 \cdot 10^{4} \cdot (2.84 \cdot 10^{-1})^2 e^{-2.84 \cdot 10^{-1} z}\left[-C_{21}\cos 2.84 \cdot 10^{-1}\right.$$

$$\left. + C_{22} \sin 2.84 \cdot 10^{-1} z\right] = -3.16 \cdot 10^{-3} e^{-2.84 \cdot 10^{-1} z}\left(-C_{21}\cos 2.84 \cdot 10^{-1} z + C_{22} \sin 2.84 \cdot 10^{-1} z\right)$$

$$u_{z_2} = C_0$$

$$\nabla_{r_2} = -2\beta\beta^3 e^{-\beta z}\left[C_{21}(\sin\beta z + \cos\beta z) + C_{22}(-\sin\beta z + \cos\beta z)\right] = -2 \cdot 1.92 \cdot 10^{4} \cdot (2.84 \cdot 10^{-1})^3 \cdot$$

$$\cdot e^{-2.84 \cdot 10^{-1} z}\left[C_{21}(\sin 2.84 \cdot 10^{-1} z + \ldots\right] = -9.08 \cdot 10^{-2} e^{-2.84 \cdot 10^{-1} z}\left[C_{21}(\sin 2.84 \cdot 10^{-1} z +\right.$$

$$\left. + \cos 2.84 \cdot 10^{-1} z) + C_{22}(-\sin 2.84 \cdot 10^{-1} z + \cos 2.84 \cdot 10^{-1} z)\right]$$

5. Stanovení $C_{11}, C_{12}, C_{21}, C_{22}, C_0, C_3, M'$

$$-1.92 \cdot 10^{4}\left(1.3 C_{11} - 0.7 \cdot \frac{1}{40^2} C_{12}\right) = 3.98 \cdot 10^{-3}$$

$$-1.92 \cdot 10^{4}\left(1.3 C_{11} - 0.7 \cdot \frac{1}{20^2} C_{12}\right) = M'$$

$$-3.16 \cdot 10^{-3}(-C_{21}) = M'$$

$$C_0 = 0$$

$$-9.08 \cdot 10^{-2}(C_{21} + C_{22}) = 0$$

$$0.5 C_{11} \cdot 20^2 + C_{12} \ln 20 + C_3 = 2.61 \cdot 10^{-2}(C_{21} + C_{22})$$

$$-C_{11} \cdot 20 - \frac{C_{12}}{20} = -2.84 \cdot 10^{-1}[-C_{21} + C_{22}]$$

$$w_2 = -\frac{v}{r_1}\left\{-\frac{1}{2\beta} e^{-\beta z}\left((C_{21} + C_{22})\cos\beta z\right.\right.$$

$$\left.\left. + (C_{21} - C_{22})\sin\beta z\right)\right\} = -\frac{0.3}{20}$$

$$\cdot\left\{-\frac{1}{2 \cdot 2.84 \cdot 10^{-1}} e^{-2.84 \cdot 10^{-1} z}\left[(\right.\right.$$

$$C_{21} + C_{22}) \cdot \cos 2.84 \cdot 10^{-1} z + (C_{21}$$

$$\left.\left. - C_{22}) \sin 2.84 \cdot 10^{-1} z\right]\right\} =$$

$$= 2.61 \cdot 10^{-2} e^{-2.84 \cdot 10^{-1} z} \cdot \left[(C_{21} + C_{22}) \cdot\right.$$

$$\left. \cdot \cos 2.84 \cdot 10^{-1} z + (C_{21} - C_{22}) \sin 2.84 \cdot 10^{-1}\right.$$

$$\begin{bmatrix} -2.5 \cdot 10^{4} & 8.4 & 0 & 0 & 0 & 0 \\ -2.5 \cdot 10^{4} & 33.6 & 0 & 0 & 0 & 5.21 \cdot 10^{-5} \\ 0 & 0 & 3.16 \cdot 10^{-3} & 0 & 0 & -1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 200 & 3 & -2.61 \cdot 10^{-2} & -2.61 \cdot 10^{-2} & 1 & 0 \\ -20 & -5 \cdot 10^{-2} & -2.84 \cdot 10^{-1} & 2.84 \cdot 10^{-1} & 6 & 0 \end{bmatrix} \begin{bmatrix} C_{11} \\ C_{12} \\ C_{21} \\ C_{22} \\ C_3 \\ M' \end{bmatrix} = \begin{bmatrix} -2.04 \cdot 10^{-4} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$C_{11} = 4.3\cdot10^{-12}$, $C_{12} = -2.92\cdot10^{-9}$, $C_{21} = 1.7\cdot10^{-6}$, $C_{22} = -1.7\cdot10^{-6}$, $C_3 = 4.3\cdot10^{-9}$, $M' = 5.39\cdot$

$$M_{r_1} = -1.92\cdot10^{4}\left(1.3\cdot4.3\cdot10^{-12} - 0.4\cdot\frac{-2.92\cdot10^{-9}}{r^2}\right) = -1.82\cdot10^{-4} - 3.92\cdot10^{-5}\frac{1}{r^2}$$

$$M_{t_1} = -1.92\cdot10^{4}\left(C_{11} + \frac{C_{12}}{r^2} + \nu\cdot\left(C_{21} - \frac{C_{12}}{r^2}\right)\right) = -1.92\cdot10^{4}\left(4.3\cdot10^{-12}\cdot1.3 + 0.4\cdot\frac{-2.92\cdot10^{-9}}{r^2}\right) =$$

$$= -1.82\cdot10^{-4} + 3.92\cdot10^{-5}\frac{1}{r^2}$$

$$M_{z_2} = 0$$

$$T_{R_2} = -9.08\cdot10^{2}e^{-2.87\cdot10^{-1}z}\left[1.7\cdot10^{-6}\sin 2.87\cdot10^{-1}z + 1.7\cdot10^{-6}\cos 2.87\cdot10^{-1}z + (-1.7\cdot10^{-6})\cdot\right.$$
$$\left.\cdot(-\sin 2.87\cdot10^{-1}z) - 1.7\cdot10^{-6}\cdot\cos 2.87\cdot10^{-1}z\right] = -3.09\cdot10^{-3}e^{-2.87\cdot10^{-1}z}\sin 2.87\cdot10^{-1}$$

$$M_{t_2} = r_1\cdot\frac{dT_{R_2}}{dz} = 20\cdot(-3.09\cdot10^{-3})\left[(-2.87\cdot10^{-1})e^{-2.87\cdot10^{-1}z}\sin 2.87\cdot10^{-1}z + e^{-2.87\cdot10^{-1}z}\cdot\right.$$
$$\left.\cdot 2.87\cdot10^{-1}\cos 2.87\cdot10^{-1}z\right] = -1.44\cdot10^{-2}e^{-2.87\cdot10^{-1}z}\left[-\sin 2.87\cdot10^{-1}z + \cos 2.87\cdot10^{-1}z\right.$$

$$M_{z_2} = -3.16\cdot10^{3}e^{-2.87\cdot10^{-1}z}\left(-1.7\cdot10^{-6}\cos 2.87\cdot10^{-1}z - 1.7\cdot10^{-6}\sin 2.87\cdot10^{-1}z\right) =$$
$$= 5.37\cdot10^{-3}e^{-2.87\cdot10^{-1}z}\left(\cos 2.87\cdot10^{-1}z + \sin 2.87\cdot10^{-1}z\right)$$

$$M_{t_2} = 0.3\cdot5.37\cdot10^{-3}e^{-2.87\cdot10^{-1}z}\left(\cos 2.87\cdot10^{-1}z - \sin 2.87\cdot10^{-1}z\right) = 1.61\cdot10^{-3}e^{-2.87\cdot10^{-1}z}\cdot$$
$$\cdot\left(\cos 2.87\cdot10^{-1}z + \sin 2.87\cdot10^{-1}z\right)$$

$$u_z = 1.7\cdot10^{-6}e^{-2.87\cdot10^{-1}z}\left(\sin 2.87\cdot10^{-1}z - \cos 2.87\cdot10^{-1}z\right)$$

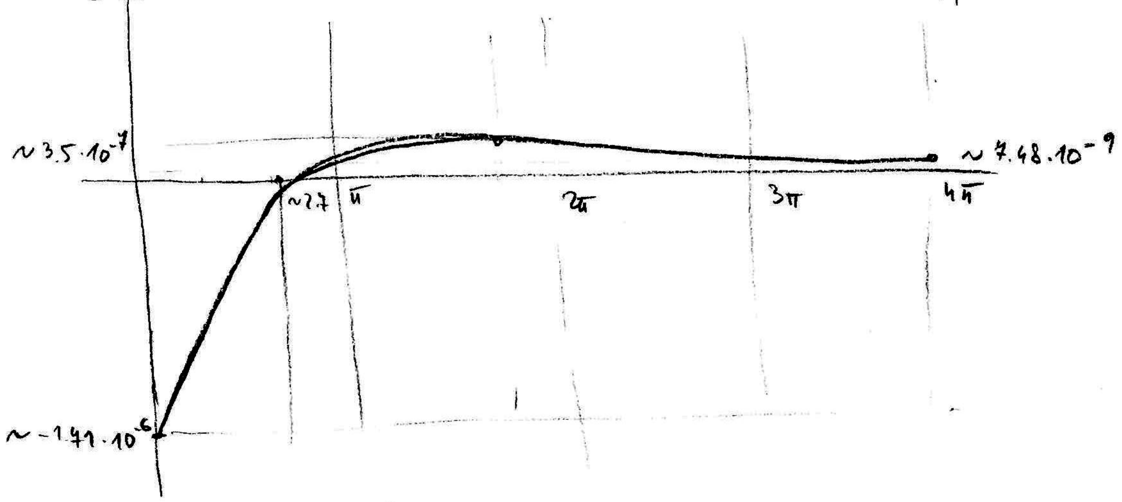
$$\sigma^1_{n,ex} = \pm\frac{6M_{r_1}}{h^2} = \pm\frac{6M_{r_1}}{1^2} = \pm 6M_{r_1}$$

$$\sigma^1_{t,ex} = \pm\frac{6M_{t_1}}{h^2} = \pm 6M_{t_1}$$

$$\sigma^2_{z,ex} = \frac{M_{z_2}}{h} \pm \frac{6M_{z_2}}{h^2} = \pm 6M_{z_2}$$

$$\sigma^2_{t,ex} = \frac{M_{t_2}}{h} \pm \frac{6M_{t_2}}{h^2} = e^{-2.87\cdot10^{-1}z}\left[\left(1.44\cdot10^{-2} \pm 1.61\cdot10^{-3}\cdot6\right)\sin 2.87\cdot10^{-1}z + \left(-1.44\cdot10^{-2} \pm\right.\right.$$
$$\left.\left.\pm 1.61\cdot10^{-3}\cdot6\right)\cos 2.87\cdot10^{-1}z\right] = e^{-2.87\cdot10^{-1}z}\left[2.44\cdot10^{-2}\sin 2.87\cdot10^{-1}z - 8.04\cdot10^{-3}\cos\right.$$
$$\left.2.87\cdot10^{-1}z\right]$$

vahová skořepina (zanedbatelné, je téměř nulové)



deska ①